An Inversion Tool for Conditional Term Rewriting Systems

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Forms of inversion: Full

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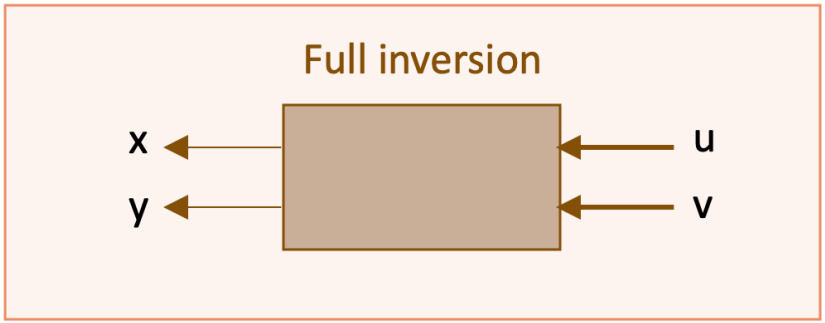
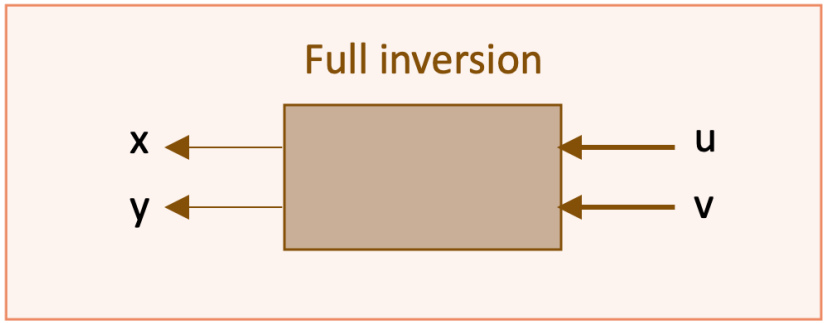
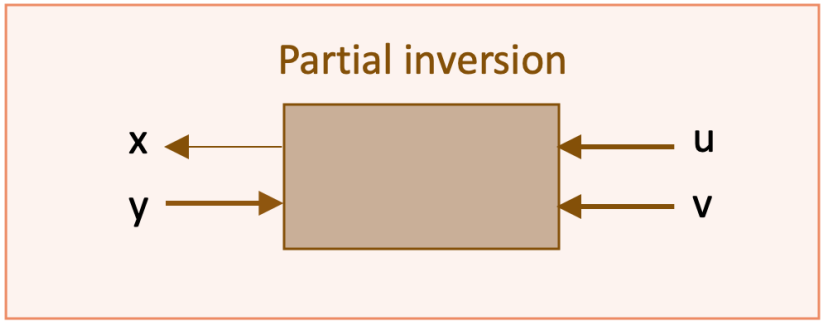
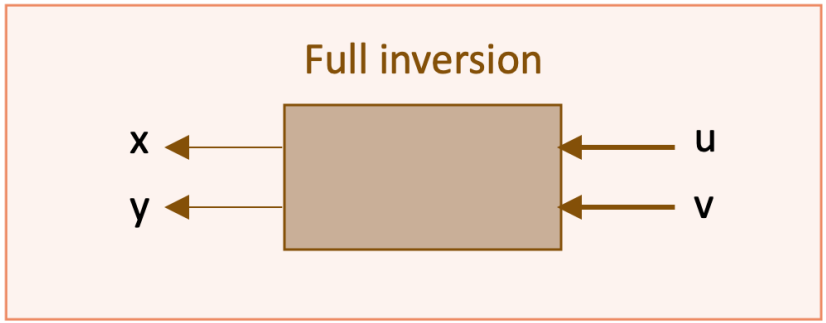


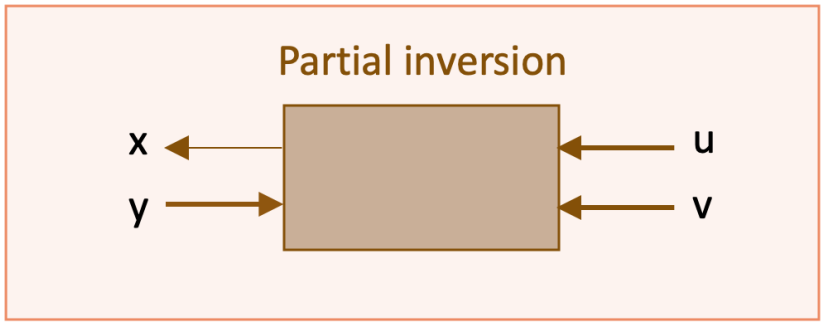
Forms of inversion: Partial (A. Y. Romanenko, 1988)

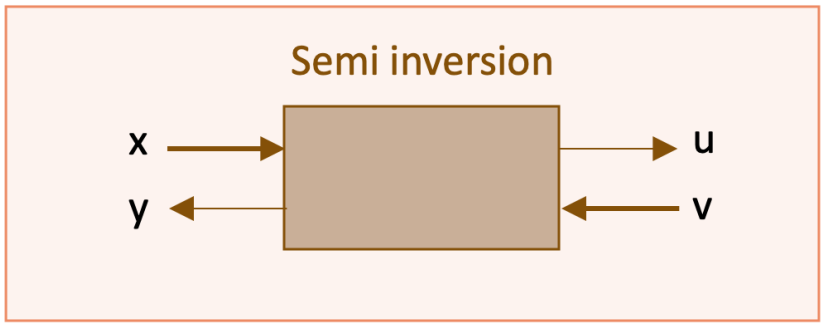




Forms of inversion: Semi







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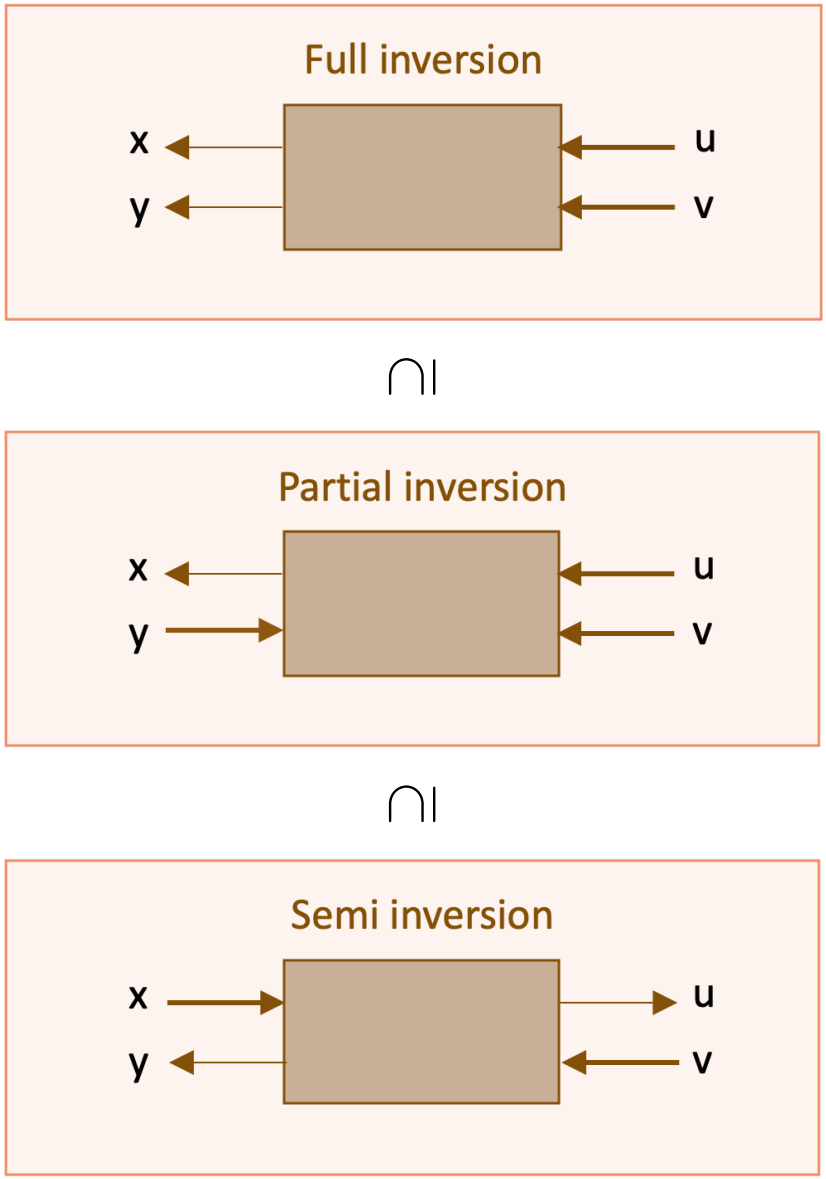

Example: Inversion of Remove Index

$$\mathsf{rem}(:(x, xs), 0) \rightarrow \langle x, xs \rangle$$
$$\mathsf{rem}(:(x, xs), s(i)) \rightarrow \langle y, :(x, zs) \rangle \Leftarrow \mathsf{rem}(xs, i) \rightarrow \langle y, zs \rangle$$

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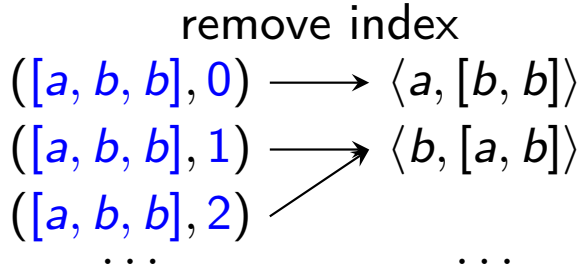
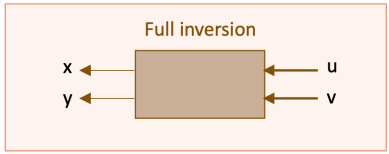


Example: Inversion of Remove Index



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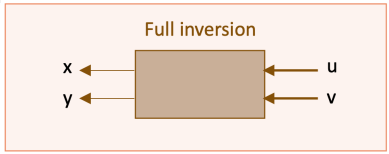
---

remove index

$$([a, b, b], 0) \longrightarrow \langle a, [b, b] \rangle$$
$$([a, b, b], 1) \longrightarrow \langle b, [a, b] \rangle \quad \xRightarrow{\text{Full inv.}}$$
$$([a, b, b], 2)$$
$$\ldots \qquad\qquad \ldots$$

Example: Inversion of Remove Index



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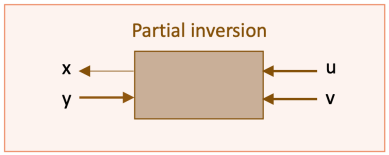
$([a, b, b], 1) \longrightarrow \langle b, [a, b] \rangle$
$([a, b, b], 2)$
$\ldots \qquad \ldots$

$\xRightarrow{\text{Full inv.}}$

random insert

$(a, [b, b])$ → $\langle [a, b, b], 0 \rangle$, $\langle [b, a, b], 1 \rangle$, $\langle [b, b, a], 2 \rangle$
$\ldots$

Example: Inversion of Remove Index



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$([a, b, b], 2)$
$\ldots \qquad \ldots$

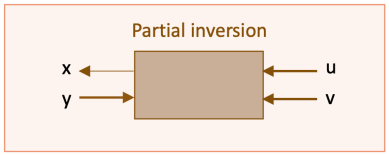
Full inv. $\Longrightarrow$

random insert

$(a, [b, b])$ → $\langle [a, b, b], 0 \rangle$, $\langle [b, a, b], 1 \rangle$, $\langle [b, b, a], 2 \rangle$
$\ldots$

Partial inv. $\Longrightarrow$

Example: Inversion of Remove Index



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$([a, b, b], 0) \longrightarrow \langle a, [b, b] \rangle$
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$\cdots \qquad \cdots$

Full inv. $\Longrightarrow$

random insert

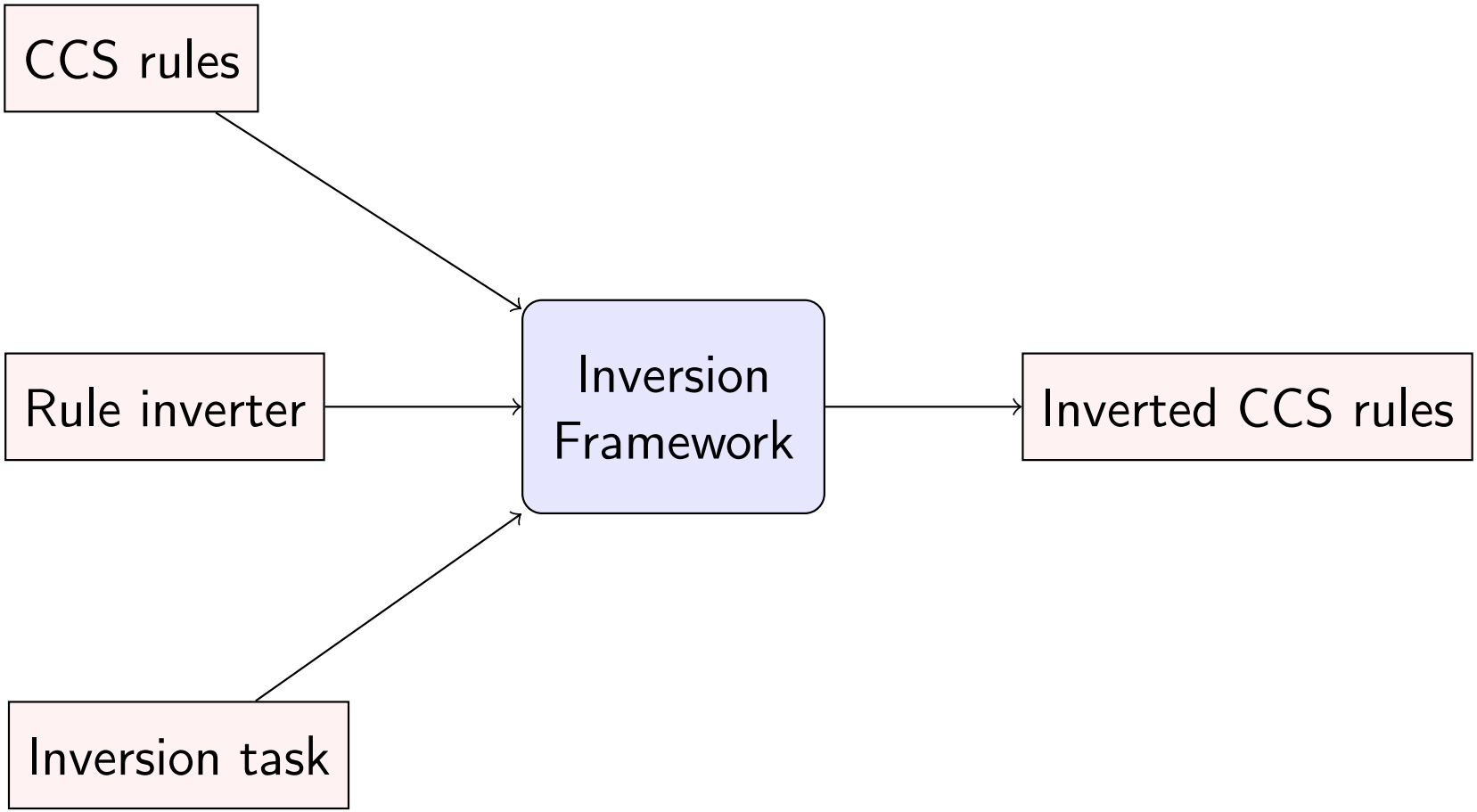
$(a, [b, b])$ → $\langle [a, b, b], 0 \rangle$, $\langle [b, a, b], 1 \rangle$, $\langle [b, b, a], 2 \rangle$, $\cdots$

Partial inv. $\Longrightarrow$

$(0, a, [b, b])$
$(1, b, [a, b])$
$(2, b, [a, b])$
$\cdots$

insert → $\langle [a, b, b] \rangle$, $\cdots$

The Generic Inversion Framework (Kirkeby & Glück, 2020)



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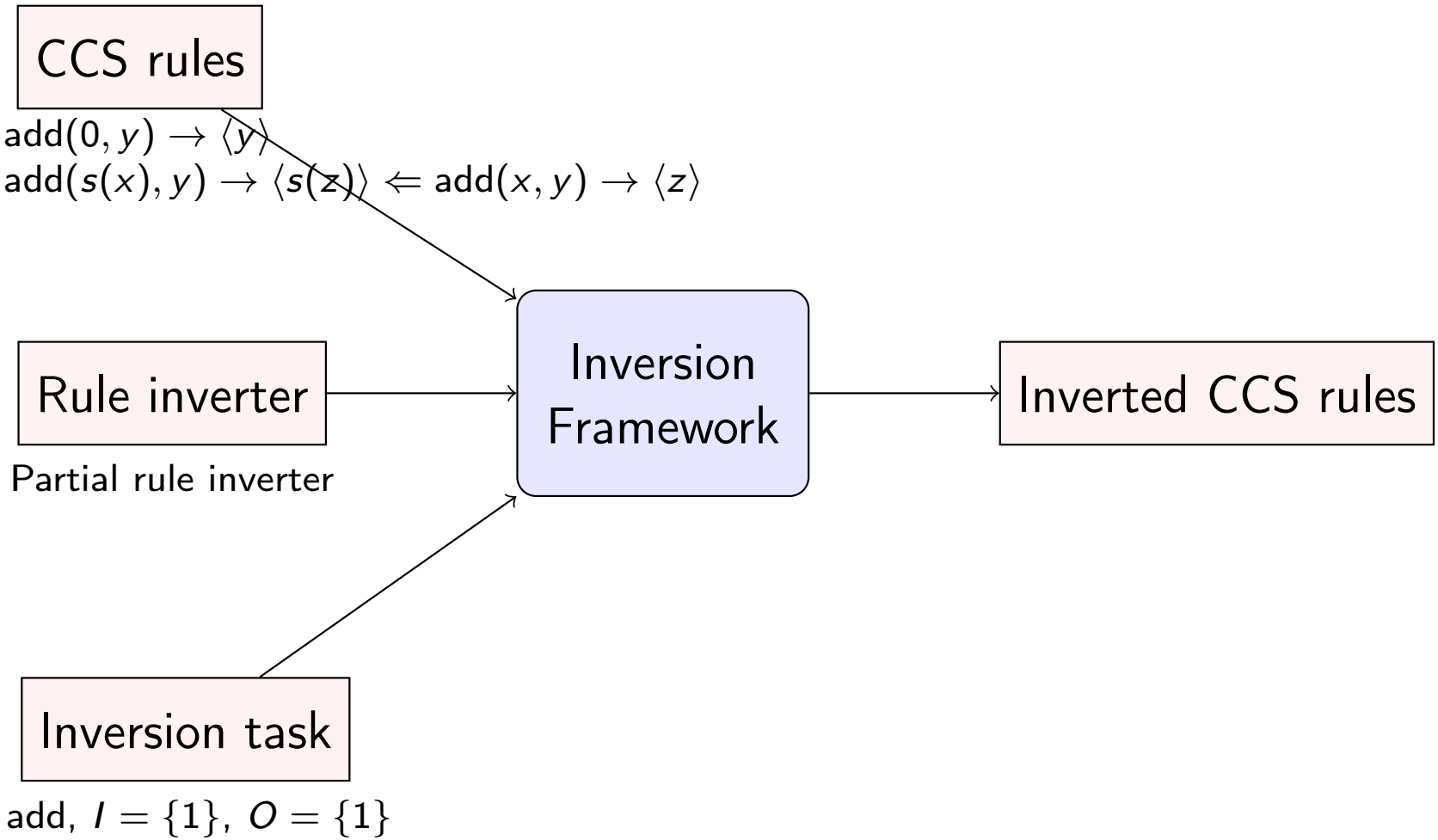

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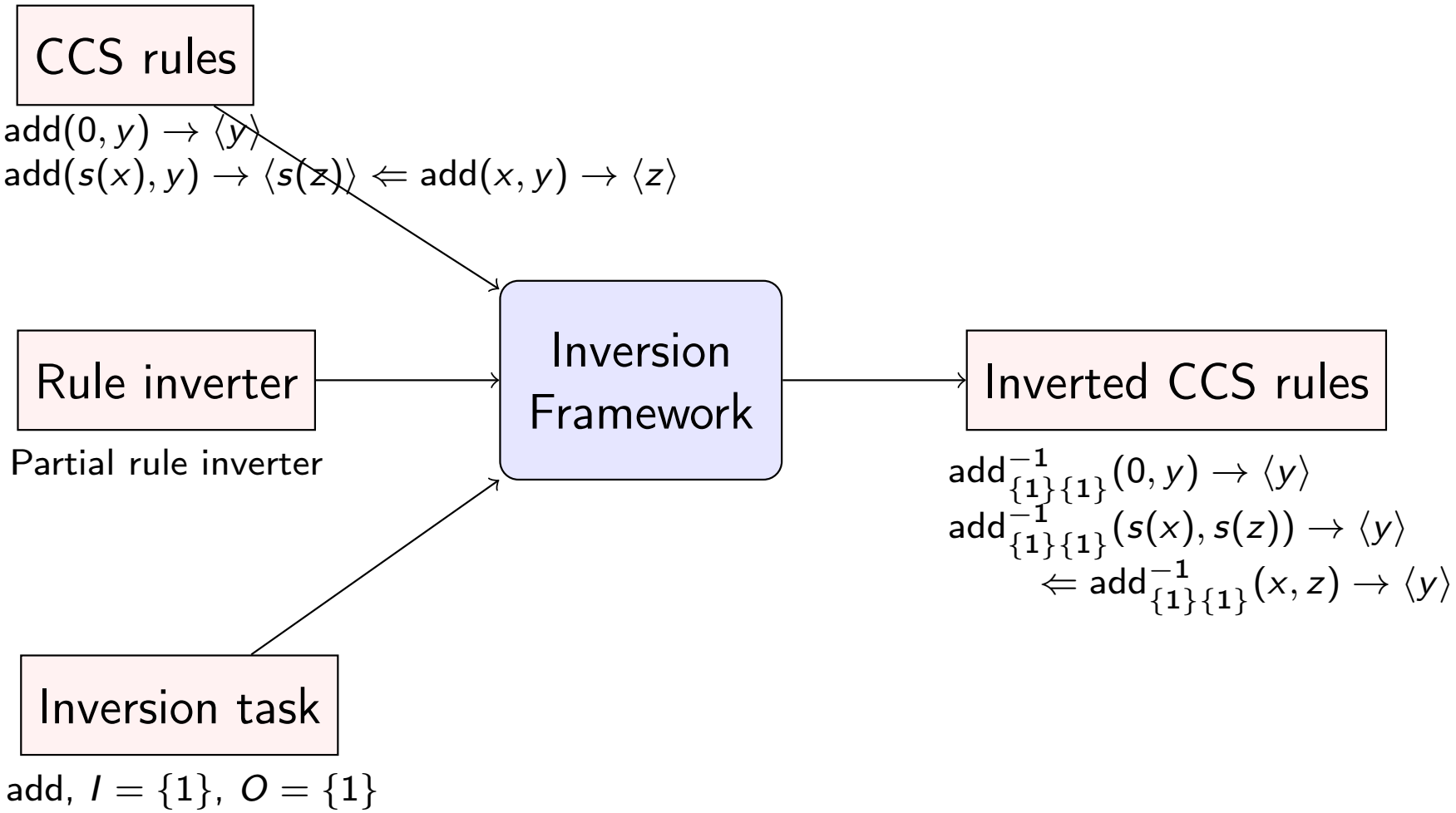

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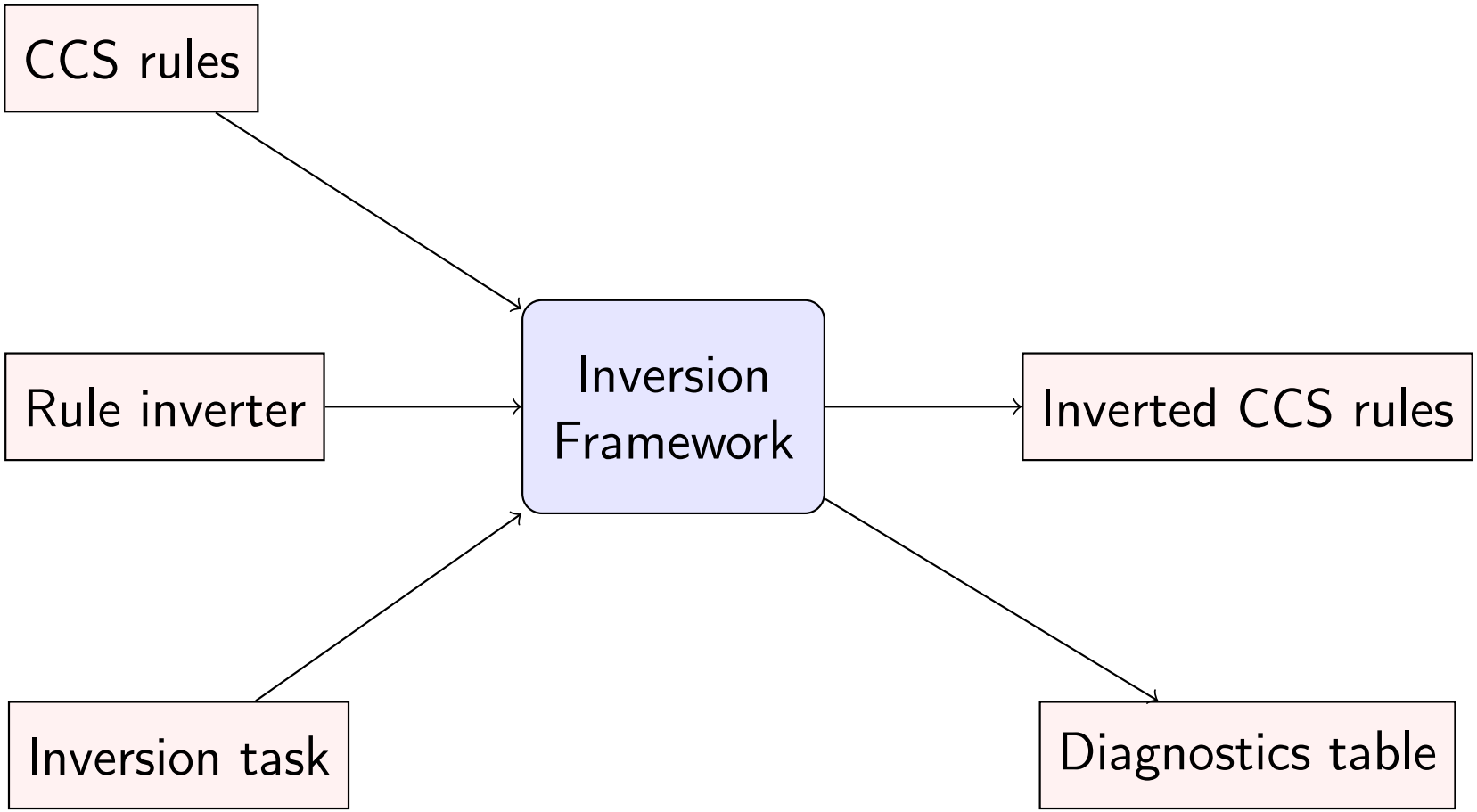

Experiment: Ackermann (A. Y. Romanenko, 1991)

$$
\begin{array}{ll}
\mathsf{ack}(0, y) & \rightarrow \langle s(y) \rangle \\
\mathsf{ack}(s(x), 0) & \rightarrow \langle z \rangle \Leftarrow \mathsf{ack}(x, s(0)) \rightarrow \langle z \rangle \\
\mathsf{ack}(s(x), s(y)) & \rightarrow \langle z \rangle \Leftarrow \mathsf{ack}(s(x), y) \rightarrow \langle v \rangle \ \wedge \mathsf{ack}(x, v) \rightarrow \langle z \rangle
\end{array}
$$

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\end{array}
$$

Demonstration time!

Experiment: Ackermann (A. Y. Romanenko, 1991)

$$
\begin{array}{lll}
\mathsf{ack}(0, y) & \to \langle s(y) \rangle & \\
\mathsf{ack}(s(x), 0) & \to \langle z \rangle \Leftarrow \mathsf{ack}(x, s(0)) \to \langle z \rangle & \\
\mathsf{ack}(s(x), s(y)) & \to \langle z \rangle \Leftarrow \mathsf{ack}(s(x), y) \to \langle v \rangle \ \wedge \mathsf{ack}(x, v) \to \langle z \rangle &
\end{array}
$$

$$\xRightarrow{\text{Partial inv.}}$$

$$
\begin{array}{lll}
\mathsf{ack}^{-1}_{\{1\}\{1\}}(0, s(y)) \to \langle y \rangle & & \\
\mathsf{ack}^{-1}_{\{1\}\{1\}}(s(x), z) \to \langle 0 \rangle & \Leftarrow \mathsf{ack}^{-1}_{\{1,2\}\{1\}}(x, s(0), z) \to \langle\rangle & \\
\mathsf{ack}^{-1}_{\{1\}\{1\}}(s(x), z) \to \langle s(y) \rangle & \Leftarrow \mathsf{ack}^{-1}_{\{1\}\{1\}}(x, z) \to \langle v \rangle \wedge & \\
 & \quad \mathsf{ack}^{-1}_{\{1\}\{1\}}(s(x), v) \to \langle y \rangle & \\
\mathsf{ack}^{-1}_{\{1,2\}\{1\}}(0, y, s(y)) \to \langle\rangle & & \\
\mathsf{ack}^{-1}_{\{1,2\}\{1\}}(s(x), 0, z) \to \langle\rangle & \Leftarrow \mathsf{ack}^{-1}_{\{1,2\}\{1\}}(x, s(0), z) \to \langle\rangle & \\
\mathsf{ack}^{-1}_{\{1,2\}\{1\}}(s(x), s(y), z) \to \langle\rangle & \Leftarrow \mathsf{ack}^{-1}_{\{1\}\{1\}}(x, z) \to \langle v \rangle \wedge & \\
 & \quad \mathsf{ack}^{-1}_{\{1,2\}\{1\}}(s(x), y, v) \to \langle\rangle &
\end{array}
$$

Experiment: Ackermann (A. Y. Romanenko, 1991)

$$
\begin{array}{lll}
\mathsf{ack}(0, y) & \rightarrow \langle s(y) \rangle & \\
\mathsf{ack}(s(x), 0) & \rightarrow \langle z \rangle \Leftarrow \mathsf{ack}(x, s(0)) \rightarrow \langle z \rangle & \\
\mathsf{ack}(s(x), s(y)) & \rightarrow \langle z \rangle \Leftarrow \mathsf{ack}(s(x), y) \rightarrow \langle v \rangle \ \wedge \mathsf{ack}(x, v) \rightarrow \langle z \rangle &
\end{array}
$$

$$\xRightarrow{\text{Partial inv.}}$$

$$
\begin{array}{lll}
\mathsf{ack2}(0, s(y)) \rightarrow \langle y \rangle & & \\
\mathsf{ack2}(s(x), z) \rightarrow \langle 0 \rangle & \Leftarrow \mathsf{ack2}(x, z) \rightarrow \langle s(0) \rangle & \\
\mathsf{ack2}(s(x), z) \rightarrow \langle s(y) \rangle & \Leftarrow \mathsf{ack2}(x, z) \rightarrow \langle v \rangle \ \wedge \mathsf{ack2}(s(x), v) \rightarrow \langle y \rangle &
\end{array}
$$

Conclusion and Future Work

- A program inversion tool for experimental and educational purposes.

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- Post-optimizations
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- Post-optimizations
  - Reduction of nondeterminism by determinization.
  - Partial evaluation of constants.
- Translating CCS into logic or functional-logic language (e.g. Prolog, Curry).

Thank you!

References

[1] Maja H. Kirkeby and Robert Glück. "Inversion framework: reasoning about inversion by conditional term rewriting systems". In: *Principles and Practice of Declarative Programming. Proceedings*. ACM, 2020, Article 9. DOI: 10.1145/3414080.3414089.

[2] Alexander Y. Romanenko. "Inversion and metacomputation". In: *Partial Evaluation and Semantics-Based Program Manipulation. Proceedings*. ACM, 1991, pp. 12–22. DOI: 10.1145/115865.115868.

[3] Alexander Y. Romanenko. "The generation of inverse functions in Refal". In: *Partial Evaluation and Mixed Computation*. Ed. by Dines Bjørner, Andrei P. Ershov, and Neil D. Jones. North-Holland, 1988, pp. 427–444.