An Empirical Study of Partial Deduction for miniKanren

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miniKanren: Relational Programming Language (Family)

embedded

relation

```
let rec appendᵒ x y r =
 ocanren {
   (x ≡ [] & y ≡ r) |
   (fresh h x' r' in
     x ≡ h :: x' &
     appendᵒ x' y r' &
     r ≡ h :: r')}
```

miniKanren: Relational Programming Language (Family)

embedded

relation

```
let rec append° x y r =
  ocanren {
    (x ≡ [] & y ≡ r) |
    (fresh h x' r' in
      x ≡ h :: x' &
      append° x' y r' &
      r ≡ h :: r')}
```

term unification

disjunction

variable introduction

conjunction

relation call

miniKanren: Querying

```
let rec appendᵒ x y r =
  ocanren {
    (x ≡ [] & y ≡ r) |
    (fresh h x' r' in
      x ≡ h :: x' &
      appendᵒ x' y r' &
      r ≡ h :: r')}
```

- **fresh** q **in** $\texttt{append}^o$ [1] [2] q
  - $\langle$ q $\to$ [1,2] $\rangle$

miniKanren: Querying

```
let rec appendᵒ x y r =
  ocanren {
    (x ≡ [] & y ≡ r) |
    (fresh h x' r' in
      x ≡ h :: x' &
      appendᵒ x' y r' &
      r ≡ h :: r')}
```

- **fresh** q **in** $\texttt{append}^o$ [1] [2] q
  - ⟨ q → [1,2] ⟩
- **fresh** x, y **in** $\texttt{append}^o$ x y [1,2]
  - ⟨ x → [], y → [1,2] ⟩
  - ⟨ x → [1], y → [2] ⟩
  - ⟨ x → [1,2], y → [] ⟩

miniKanren: Querying

```
let rec appendᵒ x y r =
  ocanren {
    (x ≡ [] & y ≡ r) |
    (fresh h x' r' in
      x ≡ h :: x' &
      appendᵒ x' y r' &
      r ≡ h :: r')}
```

- **fresh** q **in** append^o [1] [2] q
  - $\langle$ q $\rightarrow$ [1,2] $\rangle$
- **fresh** x, y **in** append^o x y [1,2]
  - $\langle$ x $\rightarrow$ [], y $\rightarrow$ [1,2] $\rangle$
  - $\langle$ x $\rightarrow$ [1], y $\rightarrow$ [2] $\rangle$
  - $\langle$ x $\rightarrow$ [1,2], y $\rightarrow$ [] $\rangle$
- **fresh** x, y, z **in** append^o x y z
  - $\langle$ x $\rightarrow$ [], y $\rightarrow$ $\_{}_0$, z $\rightarrow$ $\_{}_0$ $\rangle$
  - $\langle$ x $\rightarrow$ [$\_{}_0$], y $\rightarrow$ $\_{}_1$, z $\rightarrow$ ($\_{}_0$: $\_{}_1$) $\rangle$
  - ...

miniKanren: Semantics

- Interleaving semantics
- Complete search
  - All existing answers will be eventually found
  - It is safe to reorder within conjunctions and disjunctions
  - Reordering affects the efficiency of programs

Relational Interpreters for Search Problems

Recognizer run backwards = solver

- Write a recognizer in a functional language
- Run relational conversion to get a relational interpreter from the recognizer
- Run the relational interpreter backwards

Core issue: running relational interpreter backwards is slow

Possible solution: partial deduction

Partial Deduction: a Method to Improve Logic Programs

input program

```
let rec evalᵒ fm s r =
  fm ≡ neg x & notᵒ a r & evalᵒ x s a |
  ...
```

Partial Deduction: a Method to Improve Logic Programs

input program

```
let rec evalᵒ fm s r =
  fm ≡ neg x & notᵒ a r & evalᵒ x s a |
  ...
```

```
evalᵒ fm s true
```

known argument

Partial Deduction: a Method to Improve Logic Programs

input program

let rec $\texttt{eval}^o$ fm s r =
  fm $\equiv$ neg x & $\texttt{not}^o$ a r & $\texttt{eval}^o$ x s a |
  ...

known argument

$\texttt{eval}^o$ fm s **true**

fm $\equiv$ neg x & $\texttt{not}^o$ a **true** & $\texttt{eval}^o$ x s a |
...

Partial Deduction: a Method to Improve Logic Programs

input program

```
let rec eval^o fm s r =
  fm ≡ neg x & not^o a r & eval^o x s a |
  ...
```

known argument

```
eval^o fm s true
```

```
fm ≡ neg x & not^o a true & eval^o x s a |
...
```

```
fm ≡ neg x & eval^o x s false |
...
```

Partial Deduction: a Method to Improve Logic Programs

input program

$\texttt{let rec eval}^o\ \texttt{fm s r} =$
$\quad \texttt{fm} \equiv \texttt{neg x} \ \&\ \texttt{not}^o\ \texttt{a r} \ \&\ \texttt{eval}^o\ \texttt{x s a} \mid$
$\quad \ldots$

known argument

$\texttt{eval}^o\ \texttt{fm s}\ \textbf{true}$

$\texttt{fm} \equiv \texttt{neg x} \ \&\ \texttt{not}^o\ \texttt{a}\ \textbf{true} \ \&\ \texttt{eval}^o\ \texttt{x s a} \mid$
$\ldots$

$\texttt{fm} \equiv \texttt{neg x} \ \&\ \texttt{eval}^o\ \texttt{x s}\ \textbf{false} \mid$
$\ldots$

$\ldots$

Partial Deduction: a Method to Improve Logic Programs

input program

let rec eval^o fm s r =
  fm $\equiv$ neg x & not^o a r & eval^o x s a |
  ...

known argument

eval^o fm s **true**

fm $\equiv$ neg x & not^o a **true** & eval^o x s a |
...

fm $\equiv$ neg x & eval^o x s **false** |
...

...

output

let rec eval_true^o fm s =
  fm $\equiv$ neg x & eval_false^o x s |
  ...

let rec eval_false^o fm s =
  fm $\equiv$ neg x & eval_true^o x s |
  ...

Partial Deduction for miniKanren: Bird's-eye View

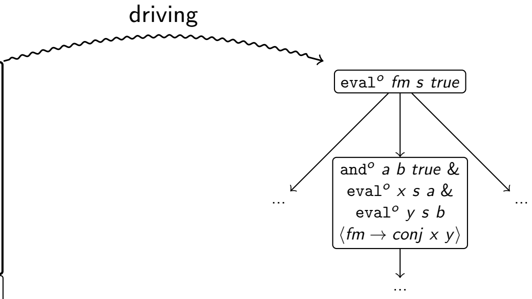
```
let rec evalᵒ fm s r =
  ...
  fm ≡ conj x y &
  andᵒ a b r &
  evalᵒ x s a &
  evalᵒ y s b |
  ...
```

```
evalᵒ fm s true
```

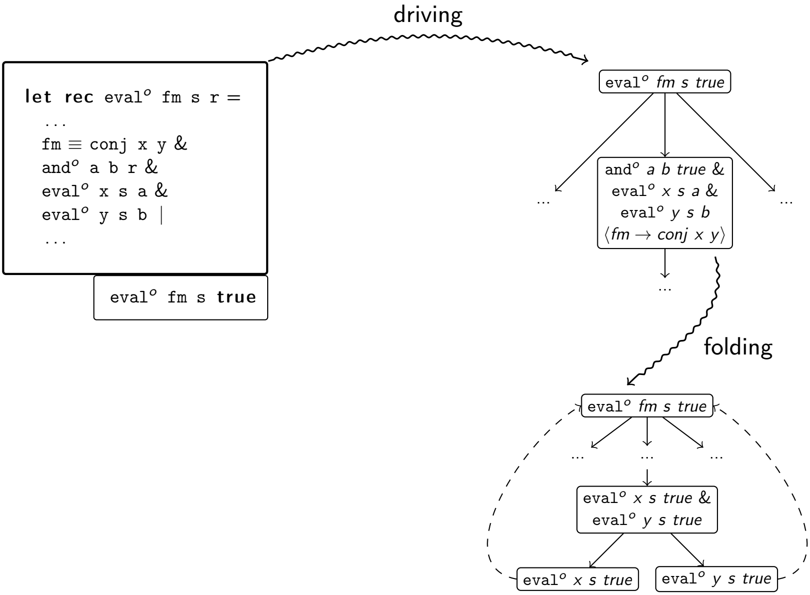
Partial Deduction for miniKanren: Bird's-eye View

```
let rec evalᵒ fm s r =
  ...
  fm ≡ conj x y &
  andᵒ a b r &
  evalᵒ x s a &
  evalᵒ y s b |
  ...
```

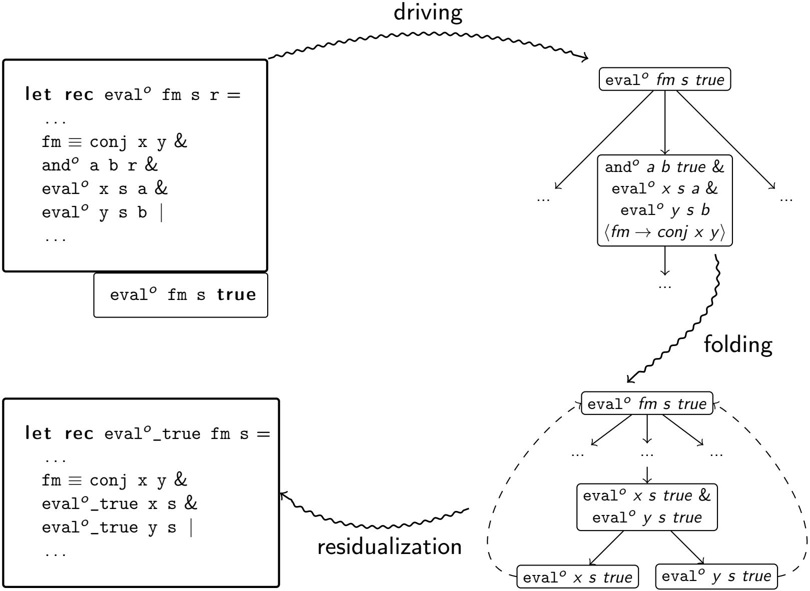
```
evalᵒ fm s true
```



Partial Deduction for miniKanren: Bird's-eye View



Partial Deduction for miniKanren: Bird's-eye View



Driving: Unfolding

```
let rec evalᵒ fm s r =
  ...
  fm ≡ conj x y & andᵒ a b r &
  evalᵒ x s a & evalᵒ y s b |
  ...

let andᵒ x y r =
  ocanren {
    fresh xy in
      (nandᵒ x y xy & nandᵒ xy xy r) }

let rec nandᵒ x y r =
  ocanren {
    (x ≡ true & y ≡ true & r ≡ false) |
    (x ≡ true & y ≡ false & r ≡ true) |
    (x ≡ false & y ≡ true & r ≡ true) |
    (x ≡ false & y ≡ false & r ≡ true) }
```

```
evalᵒ fm s true
```

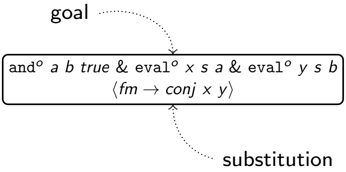
Driving: Unfolding

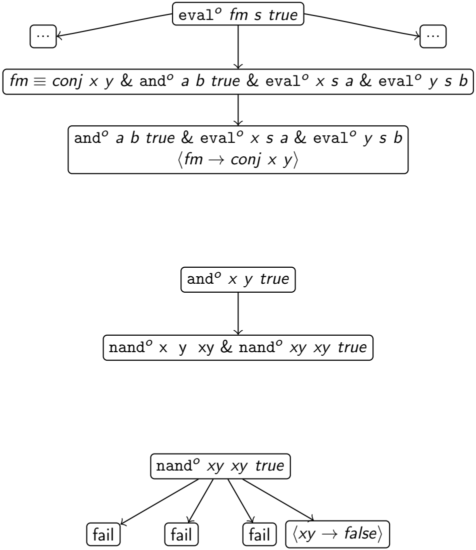
```
let rec evalᵒ fm s r =
  ...
  fm ≡ conj x y & andᵒ a b r &
  evalᵒ x s a & evalᵒ y s b |
  ...

let andᵒ x y r =
  ocanren {
    fresh xy in
      (nandᵒ x y xy & nandᵒ xy xy r) }

let rec nandᵒ x y r =
  ocanren {
    (x ≡ true & y ≡ true & r ≡ false) |
    (x ≡ true & y ≡ false & r ≡ true) |
    (x ≡ false & y ≡ true & r ≡ true) |
    (x ≡ false & y ≡ false & r ≡ true) }
```

`evalᵒ fm s true`





Partial Deduction

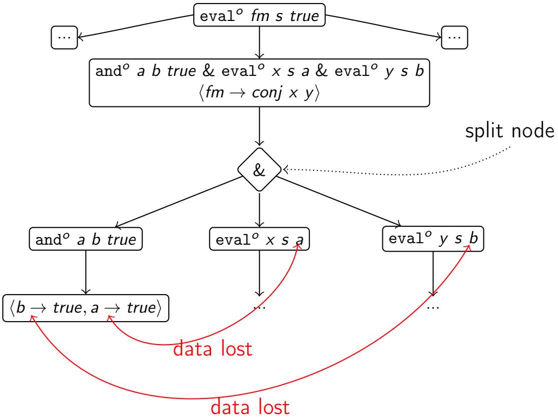
```
let rec evalᵒ fm s r =
  ...
  fm ≡ conj x y & andᵒ a b r &
  evalᵒ x s a & evalᵒ y s b |
  ...
```

```
evalᵒ fm s true
```

Partial Deduction

```
let rec evalᵒ fm s r =
  ...
  fm ≡ conj x y & andᵒ a b r &
  evalᵒ x s a & evalᵒ y s b |
  ...
```

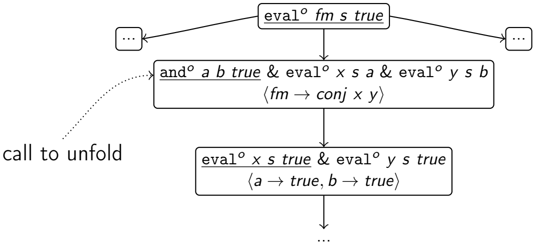
```
evalᵒ fm s true
```



Conjunctive Partial Deduction: Left-to-right Unfolding

```
let rec evalᵒ fm s r =
  ...
  fm ≡ conj x y & andᵒ a b r &
  evalᵒ x s a & evalᵒ y s b |
  ...
```

```
evalᵒ fm s true
```



CPD: Split is Necessary

```
let rec evalᵒ fm s r =
  ...
  fm ≡ conj x y & andᵒ a b r &
  evalᵒ x s a & evalᵒ y s b |
  ...
```

```
evalᵒ fm s true
```

$\underline{\texttt{eval}^o\ fm\ s\ true}$

$\dots$ $\dots$

$\underline{\texttt{and}^o\ a\ b\ true}$ & $\texttt{eval}^o\ x\ s\ a$ & $\texttt{eval}^o\ y\ s\ b$
$\langle fm \to conj\ x\ y\rangle$

$\underline{\texttt{eval}^o\ x\ s\ true}$ & $\texttt{eval}^o\ y\ s\ true$
$\langle a \to true, b \to true\rangle$

$\underline{\texttt{and}^o\ x'\ y'\ true}$ & $\texttt{eval}^o\ x'\ s\ a'$ & $\texttt{eval}^o\ y'\ s\ b'$ & $\texttt{eval}^o\ y\ s\ true$
$\langle x \to conj\ x'\ y'\rangle$

$\underline{\texttt{eval}^o\ x'\ s\ true}$ & $\texttt{eval}^o\ y'\ s\ true$ & $\texttt{eval}^o\ y\ s\ true$
$\langle a' \to true, b' \to true\rangle$

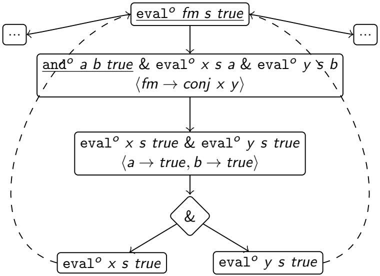
$\dots$

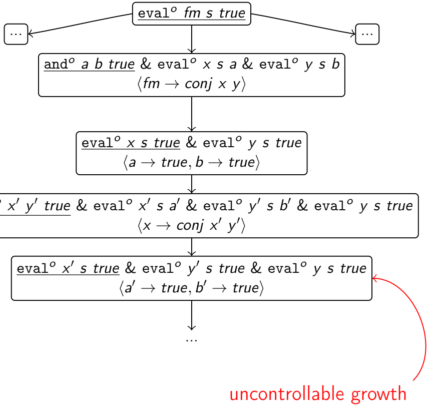
uncontrollable growth

CPD: Split is Necessary

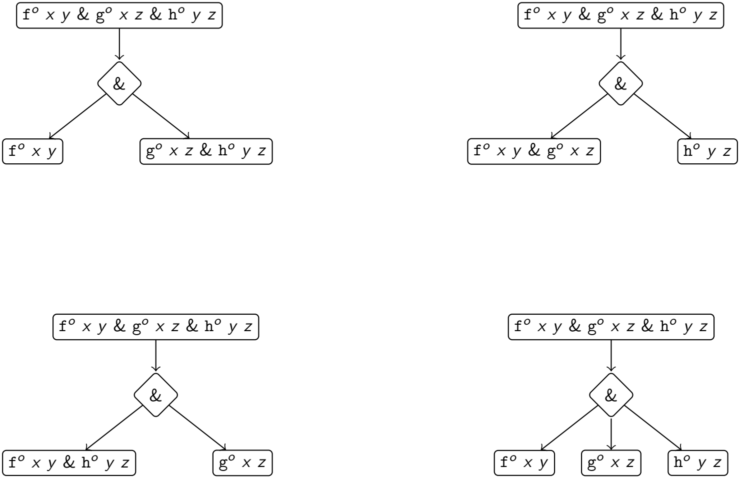
```
let rec evalᵒ fm s r =
  ...
  fm ≡ conj x y & andᵒ a b r &
  evalᵒ x s a & evalᵒ y s b |
  ...
```

`evalᵒ fm s true`





Split: Which Way is the Right Way?



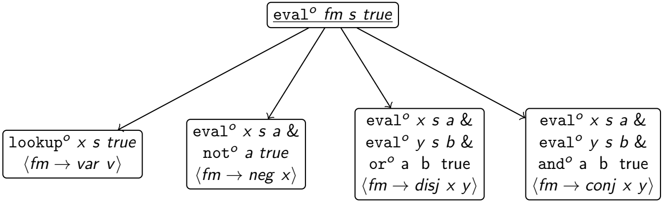
Decisions in Partial Deduction

- What to unfold: which calls, how many calls?
  - CPD: the leftmost call, which does not have a predecessor *embedded* into it
- How to unfold: to what depth a call should be unfolded?
  - CPD: unfold once
- When to stop driving?
  - When a goal is an instance of some goal in the process tree
- When to split?
  - When there is a predecessor embedded into the goal

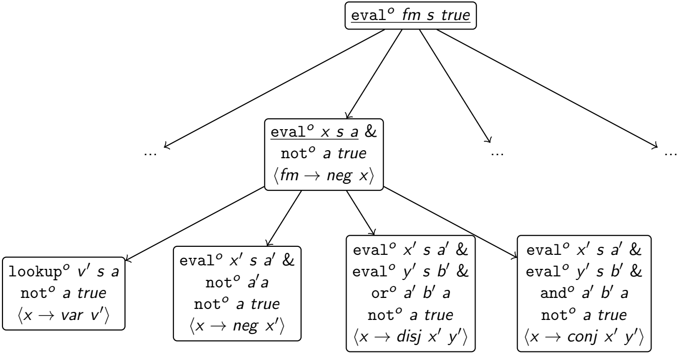
Evaluator of Logic Formulas: Unfolding Step 1

```
let rec evalᵒ fm s r =
  ocanren { fresh v x y a b in
    (fm ≡ var v & lookupᵒ v s r) |
    (fm ≡ neg x & evalᵒ x s a & notᵒ a r) |
    (fm ≡ conj x y & evalᵒ x s a & evalᵒ y s b & andᵒ a b r) |
    (fm ≡ disj x y & evalᵒ x s a & evalᵒ y s b & orᵒ a b r) }
```

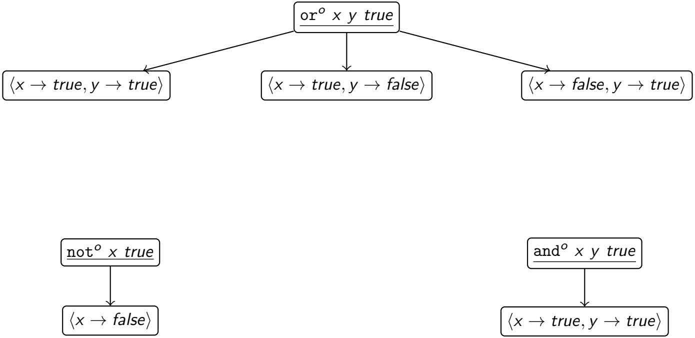
evalᵒ fm s **true**



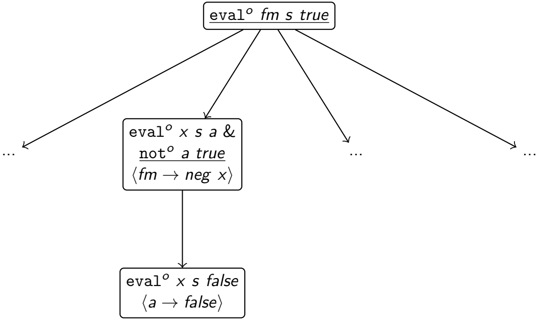
Evaluator of Logic Formulas: Unfolding Step 2



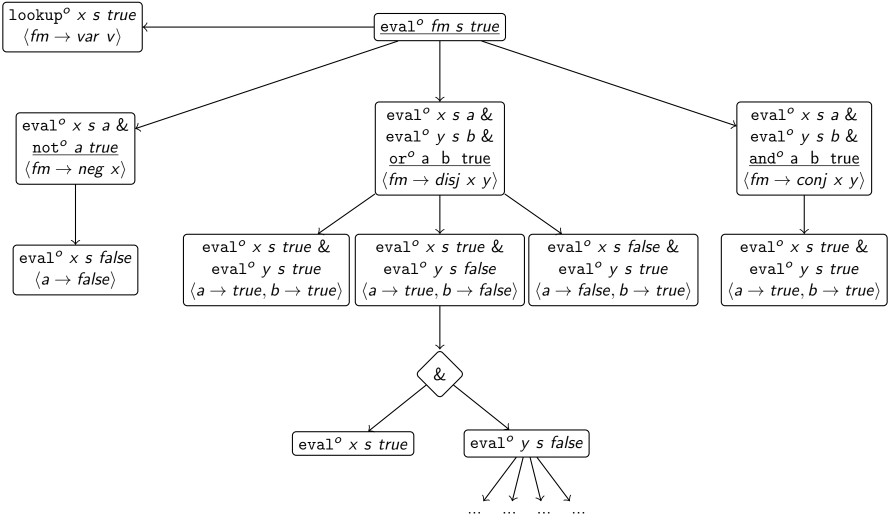
Unfolding of Boolean Connectives



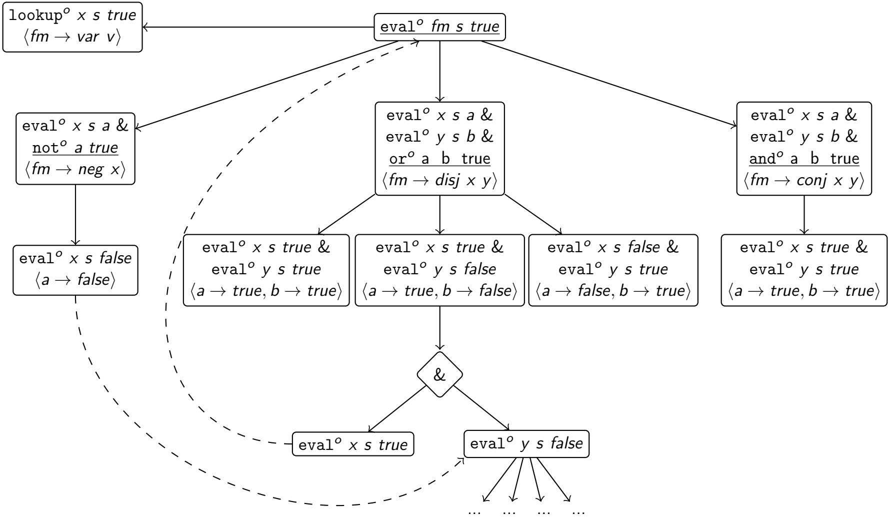
Unfolding Boolean Connectives First



Evaluator of Logic Formulas: Conservative PD



Evaluator of Logic Formulas: Conservative PD



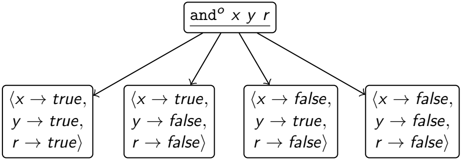
Conservative Partial Deduction

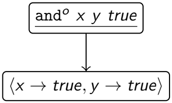
- Split conjunction into individual calls
- Unfold each call in isolation
- Unfold until embedding is encountered
- Find a call which narrows the search space (less-branching heuristics)
- Join the result of unfolding the selected call with the other calls not unfolded
- Continue driving the constucted conjunction

Less-branching Heuristics

Less-branching heuristics is used to select a call to unfold

If a call in the context unfolds into less branches than it does in isolation, select it





Evaluation

We implemented the Conservative Partial Deduction and compared it with CPD[1] on the following relations

- Four implementations of an evaluator of logic formulas
- Two implementations of a typechecker for a simple language

[1] ECCE partial deduction system

Evaluation: Comparison with ECCE

ECCE is a partial deduction system for Prolog

To compare to ECCE we did the following steps:

- Convert miniKanren program into Prolog
- Run the default transformation with ECCE
- Convert generated Prolog back into miniKanren

Evaluator of Logic Formulas: Order of Calls

boolean connective last

```
let rec evalᵒ fm s r =
  ocanren { fresh v x y a b in
    (fm ≡ var v & lookupᵒ v s r) |
    (fm ≡ neg x & evalᵒ x s a & notᵒ a r) |
    (fm ≡ conj x y & evalᵒ x s a & evalᵒ y s b & andᵒ a b r) |
    (fm ≡ disj x y & evalᵒ x s a & evalᵒ y s b & orᵒ a b r) }
```

Evaluator of Logic Formulas: Order of Calls

boolean connective last

```
let rec evalᵒ fm s r =
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    (fm ≡ conj x y & evalᵒ x s a & evalᵒ y s b & andᵒ a b r) |
    (fm ≡ disj x y & evalᵒ x s a & evalᵒ y s b & orᵒ a b r) }
```

boolean connective first

```
let rec evalᵒ fm s r =
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    (fm ≡ var v & lookupᵒ v s r) |
    (fm ≡ neg x & notᵒ a r & evalᵒ x s a) |
    (fm ≡ conj x y & andᵒ a b r & evalᵒ x s a & evalᵒ y s b) |
    (fm ≡ disj x y & orᵒ a b r & evalᵒ x s a & evalᵒ y s b) }
```

Evaluator of Logic Formulas: Compexity of Relations

table-based implementation

```
let rec andᵒ x y r =
  ocanren {
    (x ≡ true & y ≡ true & r ≡ true) |
    (x ≡ true & y ≡ false & r ≡ false) |
    (x ≡ false & y ≡ true & r ≡ false) |
    (x ≡ false & y ≡ false & r ≡ false) }
```

Evaluator of Logic Formulas: Compexity of Relations

table-based implementation

```
let rec andᵒ x y r =
  ocanren {
    (x ≡ true & y ≡ true & r ≡ true) |
    (x ≡ true & y ≡ false & r ≡ false) |
    (x ≡ false & y ≡ true & r ≡ false) |
    (x ≡ false & y ≡ false & r ≡ false) }
```

implementation via nand^o

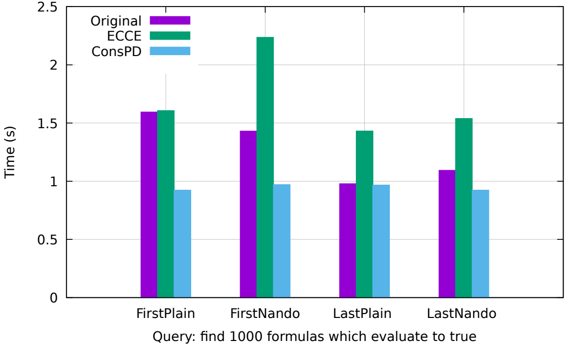
```
let andᵒ x y r =
  ocanren {
    fresh xy in
      (nandᵒ x y xy & nandᵒ xy xy r) }

let rec nandᵒ x y r =
  ocanren {
    (x ≡ true & y ≡ true & r ≡ false) |
    (x ≡ true & y ≡ false & r ≡ true) |
    (x ≡ false & y ≡ true & r ≡ true) |
    (x ≡ false & y ≡ false & r ≡ true) }
```

Evaluator of Logic Formulas: Evaluation

	Implementation	Placement
FirstPlain	table-based	before
LastPlain	table-based	after
FirstNando	via nand^o	before
LastNando	via nand^o	after

Table: Different implementations of eval^o



Typechecker-Term Generator: Language

$$
\begin{array}{rlll}
term = & BConst\ of\ Bool & \mid IConst\ of\ Int & \mid Var\ of\ Int \\
& \mid term + term & \mid term * term & \\
& \mid term = term & \mid term < term & \\
& \mid \underline{let}\ term\ \underline{in}\ term & \mid \underline{if}\ term\ \underline{then}\ term\ \underline{else}\ term &
\end{array}
$$

Figure: Language syntax

$$
\frac{}{\Gamma \vdash IConst\ i : Int} \qquad \frac{}{\Gamma \vdash BConst\ b : Bool} \qquad \frac{}{\Gamma \vdash Var\ v : \tau}\ \Gamma[v] \equiv \tau
$$

$$
\frac{\Gamma \vdash t : Int, \Gamma \vdash s : Int}{\Gamma \vdash t + s : Int} \qquad \frac{\Gamma \vdash t : \tau, \Gamma \vdash s : \tau}{\Gamma \vdash t = s : Bool} \qquad \frac{\Gamma \vdash v : \tau_v,\ (\tau_v :: \Gamma) \vdash b : \tau}{\Gamma \vdash \underline{let}\ v\ b : \tau}
$$

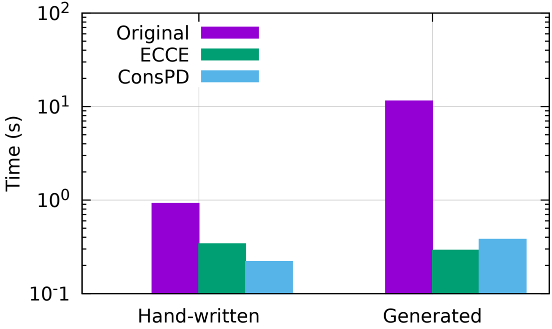
$$
\frac{\Gamma \vdash t : Int, \Gamma \vdash s : Int}{\Gamma \vdash t * s : Int} \qquad \frac{\Gamma \vdash t : Int, \Gamma \vdash s : Int}{\Gamma \vdash t < s : Bool} \qquad \frac{\Gamma \vdash c : Bool, \Gamma \vdash t : \tau, \Gamma \vdash s : \tau}{\Gamma \vdash \underline{if}\ c\ \underline{then}\ t\ \underline{else}\ s : \tau}
$$

Figure: Typing rules implemented in typecheck^o relation

Typechecker-Term Generator: Evaluation

Implementations:

- Hand-coded typing rules in MINIKANREN
- Generated from functional typechecker by relational conversion



Discussion: Order of Answers

Partial deduction changes the order of answers

Measuring time when order is different does not make much sense

Partial deduction reduces the number of unifications needed
to compute an answer

Discussion: Deterministic Unfolding and Tupling

ConsPD often splits too much failing to do tupling

Discussion: Deterministic Unfolding and Tupling

ConsPD often splits too much failing to do tupling

Because of the deterministic unfolding, ECCE fails to tuple `maxmin`

```
max([],M,M).
max([H|T],N,M) :- H =< N, max(T,N,M).
max([H|T],N,M) :- H > N, max(T,H,M).

min([],M,M).
min([H|T],N,M) :- H > N, min(T,N,M).
min([H|T],N,M) :- H =< N, min(T,H,M).

maxmin([H],H,H).
maxmin([H|T],Max,Min) :- max(T,H,Max),min(T,H,Min).
```

Conclusion

- We developed and implemented Conservative Partial Deduction
  - Less-branching heuristics
- Evaluation shows some improvement, but not for every query
- Future work:
  - Tweak ConsPD to achieve tupling in more cases
  - Develop models to predict execution time
  - Develop specialization which is more predictable, stable and well-behaved