

# Transforming Coroutining Logic Programs into Equivalent CHR Programs

Vincent Nys  
Danny De Schreye

KU Leuven

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- ▶ Variant of Compiling Control (CC)

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- ▶ Original: LP + non-standard rule  $\rightarrow$  Prolog

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## Motivation

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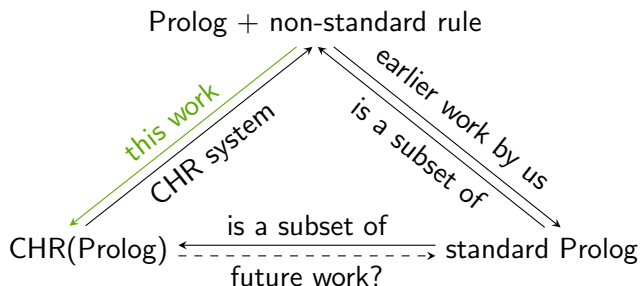
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- ▶ More natural execution model and syntax

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- ▶ More natural execution model and syntax
- ▶ Portability (CHR in C, Java, JS, Haskell,...)





red,blue  $\Leftrightarrow$  purple.

red,yellow  $\Leftrightarrow$  orange.

blue,yellow  $\Leftrightarrow$  green.

```
red,blue <=> purple.    ?- red, yellow, blue.  
red,yellow <=> orange.  S =  $\emptyset$   
blue,yellow <=> green.
```

red,blue <=> purple.      ?- yellow, blue.  
red,yellow <=> orange.     $\mathcal{S} = \{\text{red}\}$   
blue,yellow <=> green.

```
red,blue <=> purple.    ?- blue.  
red,yellow <=> orange.  S = {red, yellow}  
blue,yellow <=> green.
```

```
red,blue <=> purple.    ?- blue.  
red,yellow <=> orange.  S = {orange}  
blue,yellow <=> green.
```

```
red,blue <=> purple.    ?-.  
red,yellow <=> orange.  S = {orange, blue}  
blue,yellow <=> green.
```

## Permutation sort

```
1  sort(X,Y) :- perm(X,Y), ord(Y).
2  perm([], []).
3  perm([X|Y], [U|V]) :-
4      del(U, [X|Y], W),
5      perm(W,V).
6  ord([]).
7  ord([X]).
8  ord([X,Y|Z]) :- X =< Y, ord([Y|Z]).
```

## ACPD: absolute essentials

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- ▶ concretization function  $\gamma$

## ACPD: absolute essentials

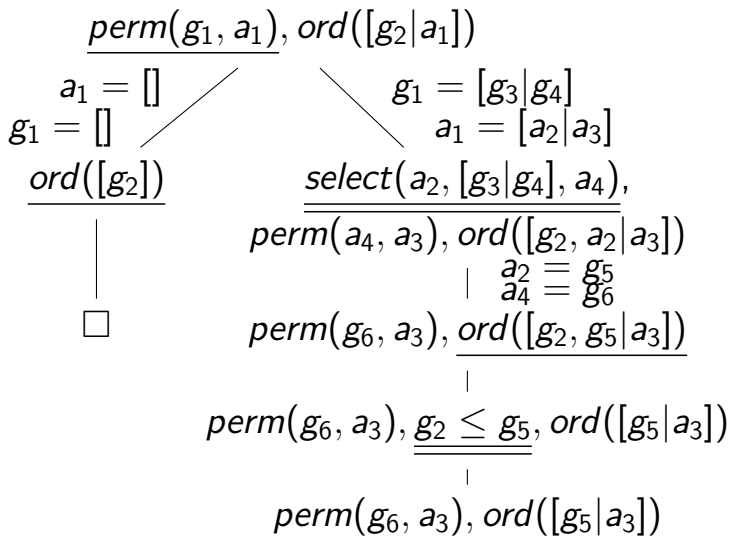
- ▶  $a_i, i \in \mathbb{N}_0$
- ▶  $g_j, j \in \mathbb{N}_0$
- ▶ abstract atoms, functions, conjunctions
- ▶ concretization function  $\gamma$
- ▶ abstract resolution

## ACPD: absolute essentials

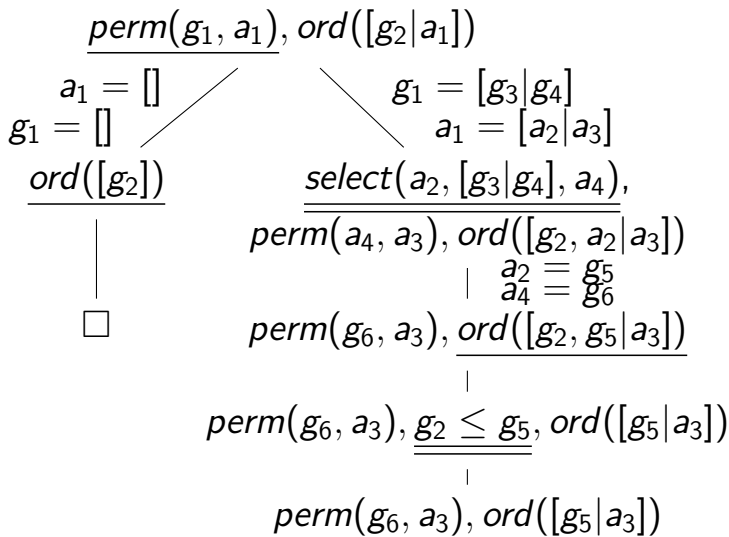
- ▶  $a_i, i \in \mathbb{N}_0$
- ▶  $g_j, j \in \mathbb{N}_0$
- ▶ abstract atoms, functions, conjunctions
- ▶ concretization function  $\gamma$
- ▶ abstract resolution
- ▶ fixpoint for  $\mathcal{A}$



## Second analysis tree



## Second analysis tree



fixpoint for  $\mathcal{A}$



# First synthesis tree, left branch

$permsort(X, Y)$



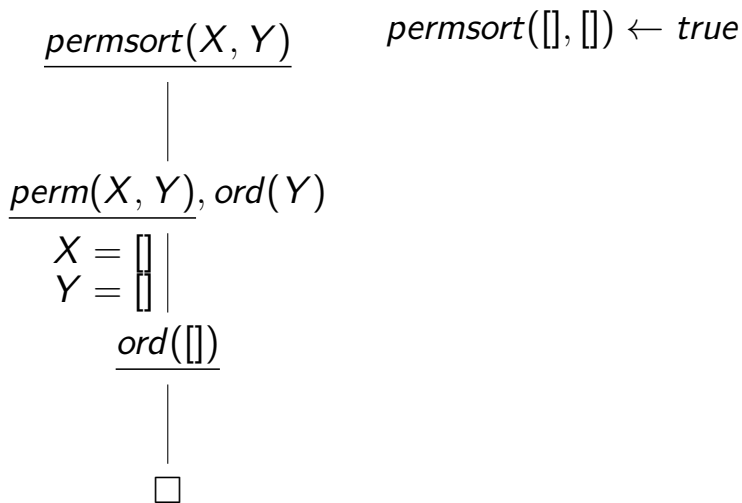
$perm(X, Y), ord(Y)$

$X = \boxed{\quad} |$   
 $Y = \boxed{\quad} |$

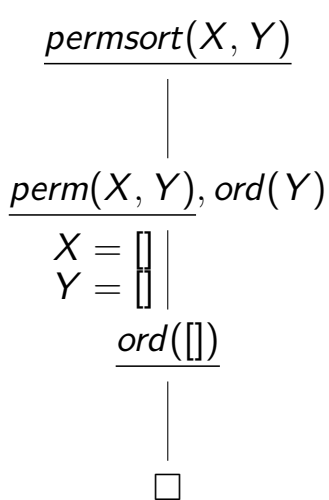
$ord(\boxed{\quad})$



# First synthesis tree, left branch

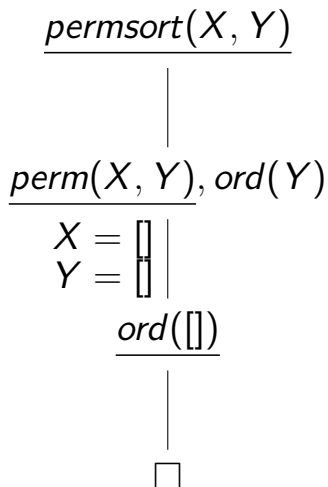


# First synthesis tree, left branch



$\text{permsort}(\square, \square) \leftarrow \text{true}$   
 $\text{permsort}(\square, \square).$

## First synthesis tree, left branch



$\text{permsort}([], []) \leftarrow \text{true}$

$\text{permsort}([], []).$

$\text{permsort}([], Y) \Leftrightarrow Y = [].$

## First synthesis tree, right branch

$permsort(X, Y)$

|

$perm(X, Y), ord(Y)$

$X = [A|B] |$   
 $Y = [C|D] |$

$select(C, [A|B], E),$   
 $perm(E, D), ord([C|D])$

|

$perm(E, D), ord([C|D])$

# First synthesis tree, right branch

$permsort(X, Y)$

|

$perm(X, Y), ord(Y)$

$X = [A|B]$  |  
 $Y = [C|D]$  |

$select(C, [A|B], E),$

$perm(E, D), ord([C|D])$

|

$perm(E, D), ord([C|D])$

$permsort([A|B], [C|D]) \leftarrow$   
 $select(C, [A|B], E) \wedge$   
 $perm(E, D) \wedge ord([C|D])$

## First synthesis tree, right branch

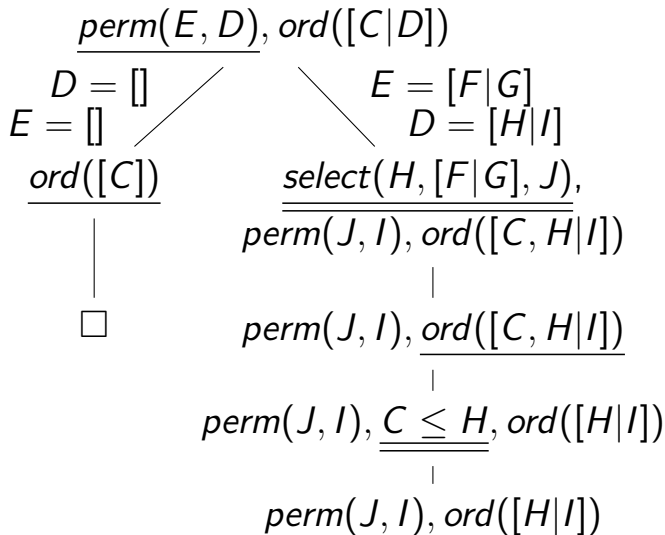
$$\begin{array}{c} \underline{\text{permsort}(X, Y)} \\ | \\ \underline{\text{perm}(X, Y), \text{ord}(Y)} \\ X = [A|B] \quad | \\ Y = [C|D] \quad | \\ \underline{\underline{\text{select}(C, [A|B], E),}} \\ \underline{\text{perm}(E, D), \text{ord}([C|D])} \\ | \\ \text{perm}(E, D), \text{ord}([C|D]) \end{array}$$
$$\begin{array}{l} \text{permsort}([A|B], [C|D]) \leftarrow \\ \text{select}(C, [A|B], E) \wedge \\ \text{perm}(E, D) \wedge \text{ord}([C|D]) \\ \\ \text{permsort}([A|B], [C|D]) :- \\ \text{select}(C, [A|B], E), \\ \text{p1}(\text{perm}(E, D), \text{ord}([C|D])). \end{array}$$

## First synthesis tree, right branch

$$\begin{array}{c} \underline{\text{permsort}(X, Y)} \\ | \\ \underline{\text{perm}(X, Y), \text{ord}(Y)} \\ X = [A|B] \mid \\ Y = [C|D] \mid \\ \underline{\text{select}(C, [A|B], E),} \\ \underline{\text{perm}(E, D), \text{ord}([C|D])} \\ | \\ \text{perm}(E, D), \text{ord}([C|D]) \end{array}$$
$$\begin{array}{l} \text{permsort}([A|B], [C|D]) \leftarrow \\ \text{select}(C, [A|B], E) \wedge \\ \text{perm}(E, D) \wedge \text{ord}([C|D]) \\ \\ \text{permsort}([A|B], [C|D]) :- \\ \text{select}(C, [A|B], E), \\ \text{p1}(\text{perm}(E, D), \text{ord}([C|D])). \\ \\ \text{permsort}([A|B], Y) \Leftrightarrow \\ Y = [C|D], \\ \text{select}(C, [A|B], E), \\ \text{perm}(E, D), \text{ord}([C|D]). \end{array}$$



## Second synthesis tree



## Confused queens

```
1  cqueens(N,D) :-
2    genlist(N,L),
3    draw(N,L,D),
4    confused(D).
5  draw(0,_,[]).
6  draw(N,L,[E|R]) :-
7    N > 0,
8    Nmin is N - 1,
9    member(E,L),
10   draw(Nmin,L,R).

1  confused([]).
2  confused([_X]).
3  confused([A,B|C]) :-
4    attack_all(A,1,[B|C]),
5    confused([B|C]).
6  attack_all(_,_,[]).
7  attack_all(A,Off,[B|C]) :-
8    Offplus is Off + 1,
9    attack(A,Off,B),
10   attack_all(A,Offplus,C).
11 attack(A,_,A).
12 attack(A,Off,B) :-
13   Diff is A - B,
14   abs(Diff,Off).
```

## Interesting branch

$draw(g_1, g_2, a_1), confused([g_3|a_1])$

|  $a_1 = [a_2|a_3]$

$g_1 > 0$ ,  $a_4$  is  $g_1 - 1$ ,  $member(a_2, g_2)$ ,

$draw(a_4, g_2, a_3), confused([g_3, a_2|a_3])$

⋮

$draw(g_4, g_2, a_3), attack\_all(g_3, g_6, [g_5|a_3]),$   
 $confused([g_5|a_3])$

| generalize

$draw(g_4, g_2, a_3), multi(attack\_all(g_3, g_6, [g_5|a_3])),$   
 $confused([g_5|a_3])$

## Interesting branch, synthesis

draw(A, B, C), confused([D|C])

| C = [E|F]

A ≥ 0, G is A - 1, member(E, B),  
draw(G, B, F), confused([D, E|F])

|  
⋮  
|

draw(G, B, F), attack\_all(D, 1, [E|F]),  
confused([E|F])

| generalize

draw(G, B, F),  
multi([attack\_all(D, H, [E|F])|I]),  
confused([E|F])

## Interesting branch, synthesis

$\frac{\text{draw}(A, B, C), \text{confused}([D C])}{\text{draw}(G, B, F), \text{confused}([D, E F])}$	$\text{draw}(A, B, C),$
$\quad \quad \quad \Big  \quad C = [E F]$	$\text{confused}([D C]) \Leftrightarrow C =$
$\frac{\underline{A \geq 0}, G \text{ is } A - 1, \text{member}(E, B),}{\text{draw}(G, B, F), \text{confused}([D, E F])}$	$[E F], A > 0, G \text{ is } A - 1,$
$\quad \quad \quad \vdots$	$\text{member}(E, B), \text{draw}(G, B, F),$
$\text{draw}(G, B, F), \text{attack\_all}(D, 1, [E F]),$	$\text{attack\_all}(D, 1, [E F]),$
$\quad \quad \quad \text{confused}([E F])$	$\text{confused}([E F]).$
$\quad \quad \quad \Big  \text{generalize}$	
$\text{draw}(G, B, F),$	
$\text{multi}([\text{attack\_all}(D, H, [E F]) I],$	
$\quad \quad \quad \text{confused}([E F])$	

## Interesting branch, synthesis

$\frac{\text{draw}(A, B, C), \text{confused}([D|C])}{\text{draw}(G, B, F), \text{confused}([D, E|F])}$

|  $C = [E|F]$

$\frac{\underline{A \geq 0}, G \text{ is } A - 1, \text{member}(E, B), \text{draw}(G, B, F), \text{confused}([D, E|F])}{\dots}$

|

$\frac{\text{draw}(G, B, F), \text{attack\_all}(D, 1, [E|F]), \text{confused}([E|F])}{\text{generalize}}$

|

$\frac{\text{draw}(G, B, F), \text{multi}([\text{attack\_all}(D, H, [E|F])|I]), \text{confused}([E|F])}{\dots}$

$\text{draw}(A, B, C),$   
 $\text{confused}([D|C]),$  **lock**  $\Leftrightarrow$   
 $C = [E|F], A > 0, G \text{ is } A - 1,$   
 $\text{member}(E, B), \text{draw}(G, B, F),$   
 $\text{attack\_all}(D, 1, [E|F]),$   
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## Interesting branch, synthesis

$\frac{\text{draw}(A, B, C), \text{confused}([D|C])}{\text{draw}(G, B, F), \text{confused}([D, E|F])}$

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|

$\frac{\text{draw}(G, B, F), \text{attack\_all}(D, 1, [E|F]), \text{confused}([E|F])}{\text{generalize}}$

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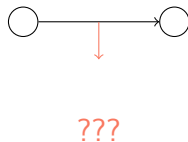
$\text{draw}(A, B, C),$   
 $\text{confused}([D|C]),$  **lock**  $\Leftrightarrow$   
 $C = [E|F], A > 0, G \text{ is } A - 1,$   
 $\text{member}(E, B), \text{draw}(G, B, F),$   
 $\text{attack\_all}(D, 1, [E|F]),$   
 $\text{confused}([E|F]),$  **lock**.



## Interesting branch, synthesis

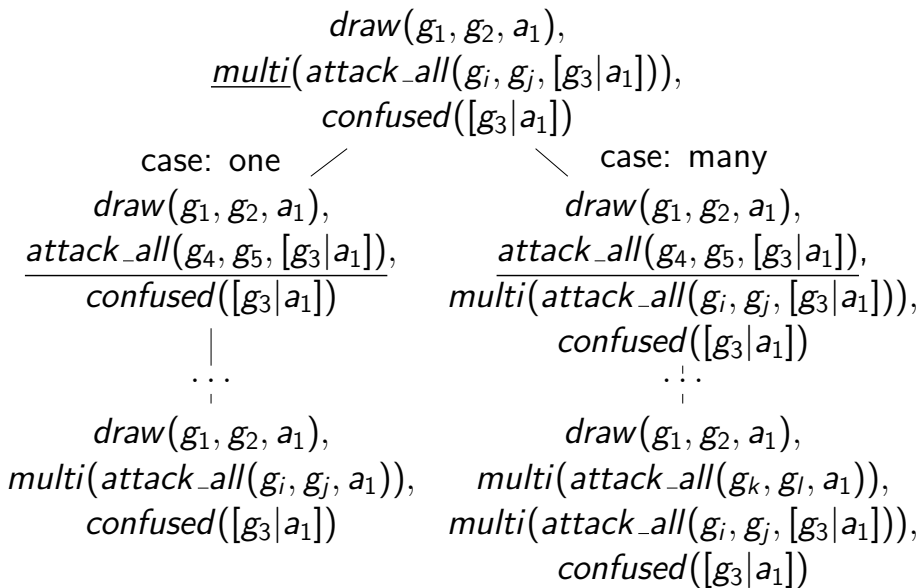
$\frac{\text{draw}(A, B, C), \text{confused}([D|C])}{\text{draw}(G, B, F), \text{confused}([D, E|F])}$   
|  $C = [E|F]$   
 $\frac{\underline{A \geq 0}, G \text{ is } A - 1, \text{member}(E, B), \text{draw}(G, B, F), \text{confused}([D, E|F])}{\dots}$   
|  
 $\text{draw}(G, B, F), \text{attack\_all}(D, 1, [E|F]), \text{confused}([E|F])$   
| generalize  
 $\text{draw}(G, B, F), \text{multi}([\text{attack\_all}(D, H, [E|F])|I]), \text{confused}([E|F])$

$\text{draw}(A, B, C), \text{confused}([D|C]), \text{lock} \Leftrightarrow C = [E|F], A > 0, G \text{ is } A - 1, \text{member}(E, B), \text{draw}(G, B, F), \text{attack\_all}(D, 1, [E|F]), \text{confused}([E|F]), \text{lock}.$





## Branches with identical computations



## Soundness?

?- cqueens(4,X).

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```

```
X = [1, 1, 1, 1] ;
```

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```
X = [1, 1, 1, 1] ;
```

```
X = [1, 2, 3, 4] ;
```

## Soundness?

```
?- cqueens(4,X).
```

```
X = [1, 1, 1, 1] ;
```

```
X = [1, 2, 3, 4] ;
```

```
X = [2, 2, 2, 2] ;
```

## Soundness?

```
?- cqueens(4,X).
```

```
X = [1, 1, 1, 1] ;
```

```
X = [1, 2, 3, 4] ;
```

```
X = [2, 2, 2, 2] ;
```

```
X = [3, 3, 3, 3] ;
```

## Soundness?

```
?- cqueens(4,X).
```

```
X = [1, 1, 1, 1] ;
```

```
X = [1, 2, 3, 4] ;
```

```
X = [2, 2, 2, 2] ;
```

```
X = [3, 3, 3, 3] ;
```

```
X = [4, 3, 2, 1] ;
```

## Soundness?

```
?- cqueens(4,X).
```

```
X = [1, 1, 1, 1] ;
```

```
X = [1, 2, 3, 4] ;
```

```
X = [2, 2, 2, 2] ;
```

```
X = [3, 3, 3, 3] ;
```

```
X = [4, 3, 2, 1] ;
```

```
X = [4, 4, 4, 4]
```



## Soundness?

```
?- cqueens(4,X).
```

```
X = [1, 1, 1, 1] ;
```

```
X = [1, 2, 3, 4] ;
```

```
X = [2, 2, 2, 2] ;
```

```
X = [3, 3, 3, 3] ;
```

```
X = [4, 3, 2, 1] ;
```

```
X = [4, 4, 4, 4]
```

```
?- cqueens(4,[A,B,C,D]).
```

## Soundness?

```
?- cqueens(4,X).
```

```
X = [1, 1, 1, 1] ;
```

```
X = [1, 2, 3, 4] ;
```

```
X = [2, 2, 2, 2] ;
```

```
X = [3, 3, 3, 3] ;
```

```
X = [4, 3, 2, 1] ;
```

```
X = [4, 4, 4, 4]
```

```
?- cqueens(4,[A,B,C,D]).
```

```
A=B, B=C, C=D, D=2 ;
```

## Soundness?

```
?- cqueens(4,X).  
X = [1, 1, 1, 1] ;  
X = [1, 2, 3, 4] ;  
X = [2, 2, 2, 2] ;  
X = [3, 3, 3, 3] ;  
X = [4, 3, 2, 1] ;  
X = [4, 4, 4, 4]
```

```
?- cqueens(4,[A,B,C,D]).  
A=B, B=C, C=D, D=2 ;  
ERROR: is/2: Arguments  
are not sufficiently  
instantiated
```

## The culprit

$draw(g_1, g_2, a_1), \underline{multi}(attack\_all(g_i, g_j, [g_3|a_1])),$   
 $confused([g_3|a_1])$

|  
⋮  
|

$draw(g_1, g_2, a_1), multi(attack\_all(g_i, g_j, a_1)),$   
 $confused([g_3|a_1])$

$lock, attack\_all(D, E, [F|C]) \Leftrightarrow$

$H \text{ is } E + 1, attack(D, E, F),$

$attack\_all(D, H, C), lock.$

## The culprit

$draw(g_1, g_2, a_1), \underline{multi}(attack\_all(g_i, g_j, [g_3|a_1])),$   
 $confused([g_3|a_1])$

⋮

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```
lock, attack_all(D,E,[F|C]) <=> H is E + 1,  
attack(D,E,F), attack_all(D,H,C), lock.
```



## The solution

- ▶ encode *assumed* level of instantiation in constraints
- ▶ atomically renaming conjunctions (in Prolog) does this *implicitly*

```
lock, attack_all(D,E,[F|C],[g,g,[g|a]]) <=>  
H is E + 1, attack(D,E,F),  
attack_all(D,H,C,[g,g,a]), lock.
```

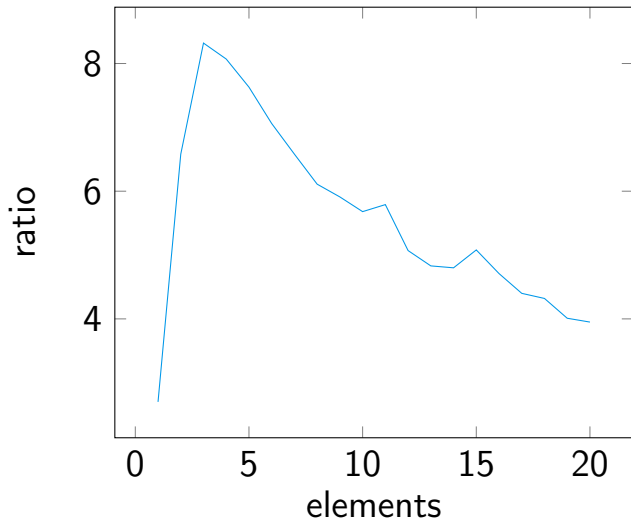
# Code (Prolog)

```
genlist(N,L) :- N >= 1, genlist_acc(N,[],L).
genlist_acc(N,Acc,L) :- N > 1, Nmin is N-1, genlist_acc(Nmin,[N|Acc],L).
genlist_acc(N,Acc,[_|Acc]) :- N is 1.
attack(A,_,A).
attack(A,Offset,B) :-
    Diff is A - B,
    abs(Diff,Offset).
queens(0,[]).
queens(A,[D|E]) :-
    genlist(A,C),
    A > 0,
    F is A - 1,
    member(D,C),
    a(draw(F,C,E),confused([D|E])).
a(draw(0,_,[]),confused(_)).
a(draw(A,B,[E|F]),confused([D,E|F])) :-
    A >= 0,
    G is A - 1,
    member(E,B),
    b(draw(G,B,F),
        multi([attack_all(D,1,[E|F])],
            confused([E|F]))).
b(draw(A,B,C),multi([attack_all(D,E,[F|C])],confused([F|C])) :-
    H is E + 1,
    attack(D,E,F),
    c(draw(A,B,C),
        multi([attack_all(D,H,C)],
            confused([F|C]))).
b(draw(A,B,C),multi([attack_all(D,E,[F|C])],
    attack_all(H,I,[F|C]|J)),confused([F|C])) :-
    K is E + 1,
    attack(D,E,F),
    d(draw(A,B,C),multi([attack_all(D,K,C)],
        multi([attack_all(H,I,[F|C]|J)),confused([F|C]))).
d(draw(A,B,C),multi([attack_all(D,E,C)|F]),
    multi([attack_all(G,H,[I|C])],confused([I|C])) :-
    K is H + 1,
    attack(G,H,I),
    append([attack_all(D,E,C)|F],[attack_all(G,K,C)],
        Appended),
    c(draw(A,B,C),multi(Appended),
        multi([attack_all(K,L,[I|C]|M)],confused([I|C]))).
e(multi([attack_all(A,B,[])],confused([Z]))).
e(multi([attack_all(A,B,[])],
    attack_all(C,D,[]|E)),confused([Z])) :-
    e(multi([attack_all(C,D,[]|E)],confused([Z])).
d(draw(A,B,C),multi([attack_all(D,E,C)|F]),
    multi([attack_all(G,H,[I|C])],attack_all(K,L,[I|C]|M)),
    confused([I|C])) :-
    N is H + 1,
    attack(G,H,I),
    append([attack_all(D,E,C)|F],[attack_all(G,N,C)],
        Appended),
    d(draw(A,B,C),multi(Appended),
        multi([attack_all(K,L,[I|C]|M)],confused([I|C]))).
c(draw(0,B,[]),multi([attack_all(D,E,[])|F]),confused([G])) :-
    e(multi([attack_all(D,E,[])|F]),confused([G])).
c(draw(A,B,[H|I]),multi([attack_all(D,E,[H|I])|F]),
    confused([G,H|I])) :-
    A > 0,
    J is A - 1,
    member(H,B),
    append([attack_all(D,E,[H|I])|F],
        [attack_all(G,1,[H|I])],Appended),
    b(draw(J,B,I),multi(Appended),confused([H|I])).
e(multi([attack_all(A,B,[])],confused([Z]))).
e(multi([attack_all(A,B,[])],
    attack_all(C,D,[]|E)),confused([Z])) :-
    e(multi([attack_all(C,D,[]|E)],confused([Z])).
```

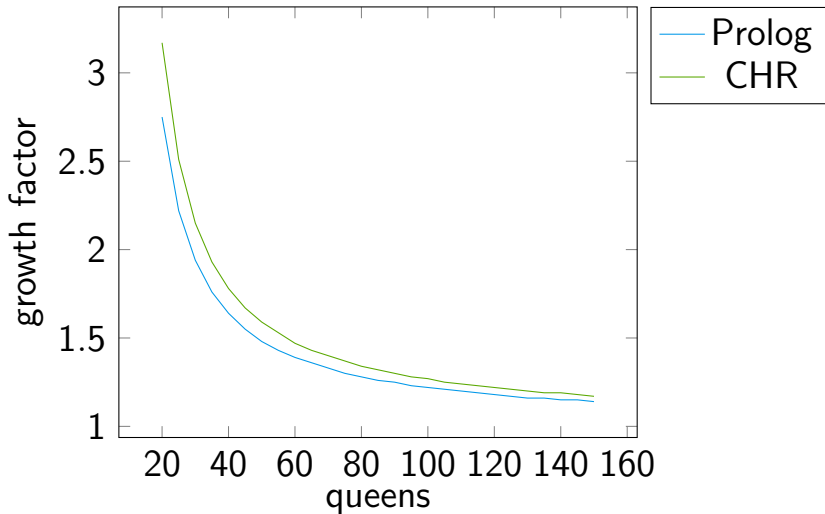
# Code (CHR)

```
genlist(N,L) :- N >= 1, genlist_acc(N,[],L).
genlist_acc(N,Acc,L) :- N > 1, Nmin is N-1, genlist_acc(Nmin,[N|Acc],L).
genlist_acc(N,Acc,[1|Acc]) :- N is 1.
attack(A,_,A).
attack(A,Offset,B) :- Diff is A - B, abs(Diff,Offset).
lock, rename, attack_all(A,B,C,[g,g,a]) <=>
  attack_all(A,B,C,[g,g,[g|a]]), rename, lock.
lock, rename <=> lock.
cqueens(O,B,[g,a]) <=> genlist(O,C), B = [].
cqueens(A,B,[g,a]) <=>
  B = [E|F],
  genlist(A,C),
  A > 0,
  D is A-1,
  member(E,C),
  draw(D,C,F,[g,g,a]),
  confused([E|F],[g|a]),
  lock.
attack_all(A,Off,[B|C],[g,g,[g|a]]), lock <=>
  Off1 is Off + 1, attack(A,Off,B), attack_all(A,Off1,C,[g,g,a]), lock.
attack_all(X,Y,[],_) , lock <=> lock.
draw(O,B,C,[g,g,a]), confused([D|C],[g|a]), lock <=> C = [], lock.
draw(A,B,C,[g,g,a]), confused([D|C],[g|a]), lock <=>
  C = [E|F],
  A>0,
  G is A-1,
  member(E,B),
  draw(G,B,F,[g,g,a]),
  rename,
  attack_all(D,1,[E|F],[g,g,[g|a]]),
  confused([E|F],[g|a]),
  lock.
lock <=> true.
```

Permutation sort:  $\frac{\text{inferences CHR}}{\text{inferences Prolog}}$



Queens:  $\frac{\text{inferences } cqueens(N,X)}{\text{inferences } cqueens(N-1,X)}$



- ▶ Concise representation

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- ▶ Portability to other CHR systems

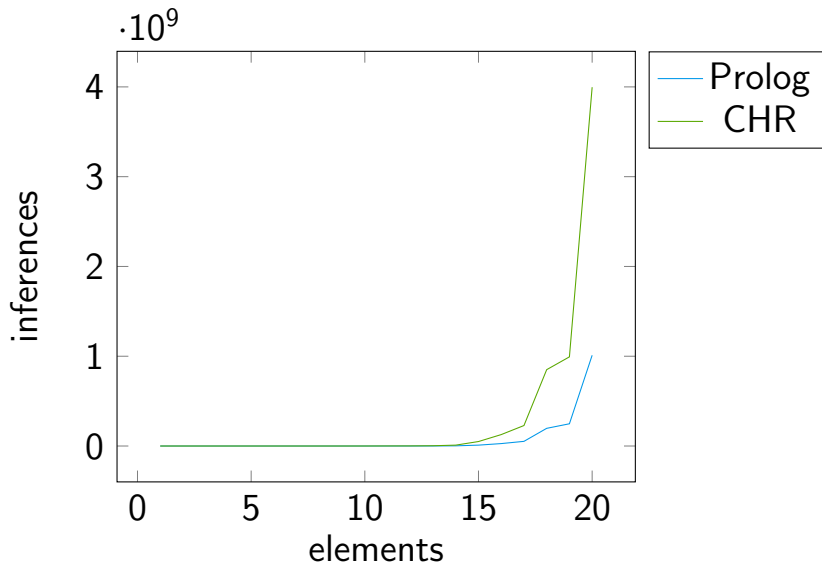


# Results

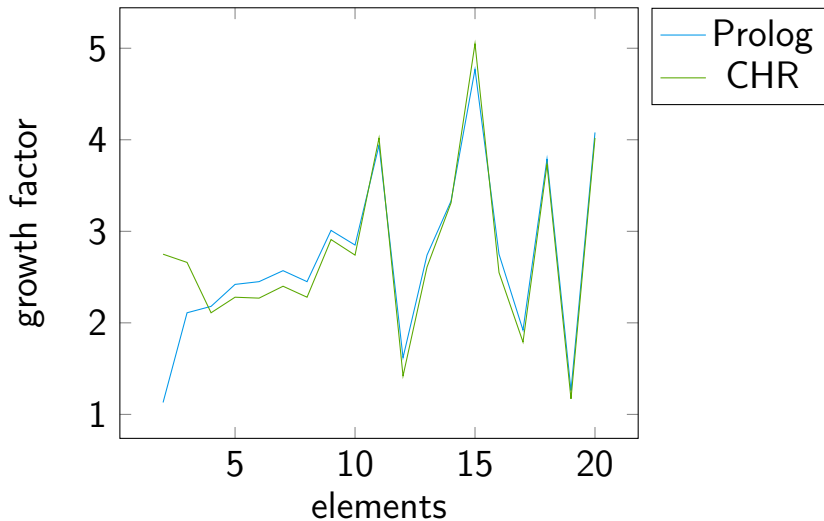
- ▶ Concise representation
- ▶ Extra target for analysis
- ▶ Portability to other CHR systems
- ▶ Slower resulting programs, but potentially same asymptotic time complexity

Questions?

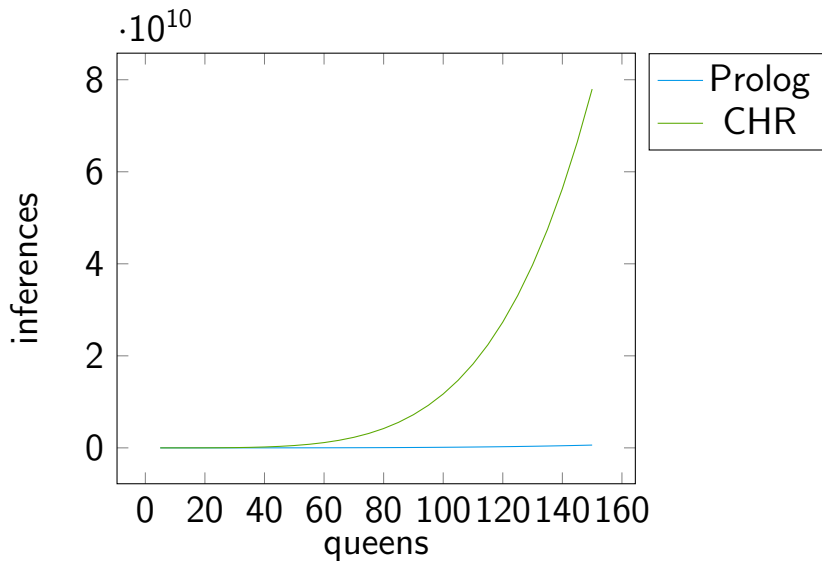
# Permutation sort: inferences



# Permutation sort: growth w.r.t. previous size



# Queens: inferences



Queens:  $\frac{\text{inferences CHR}}{\text{inferences Prolog}}$

