Energy Consumption Analysis and Verification by Translation to Horn Clauses and Abstract Interpretation

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Analysis/Debugging/Verification of Resources

Statically and automatically infer upper/lower bounds on the usage that a program makes of a general notion of (*user-definable*) resources.

- Memory, execution time, execution steps, data sizes, ...
- Bits sent/received over a socket, SMSs, database accesses, procedure calls, files left open, money spent, **energy consumed**, . . .

- Key observations about resource consumption:
  - Undecidable → infer safe **bounds** (as accurately as possible) → **AI**
  - Dependent on input data metrics → infer the bounds **as functions** of input data sizes (list length, array dimensions, numerical values, ...). (Difference with WCET and related methods.)

- Applications: performance debugging and verification, resource-oriented optimization, granularity control in parallelism, . . .

[DLH90, DLGHL94, DLGHL97, NMLGH07, MLGCH08, NMLH08, NMLH09, LGDB10, SLBH13, LKSGL13, SLH14]

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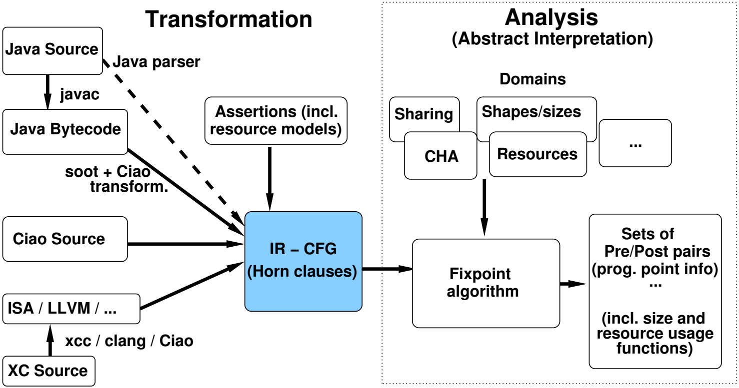
CiaoPP Intermediate Repr.: (Constraint) Horn Clauses

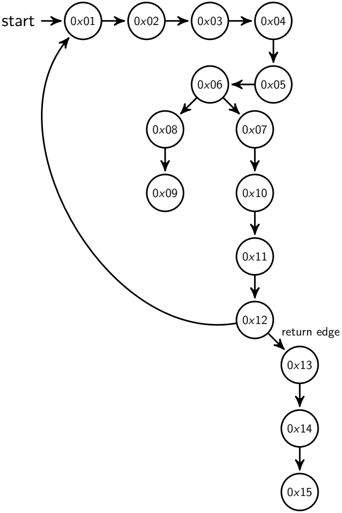
[MLNH07]



- Transformation:
  - Source: Program P in L_P + (possibly abstract) Semantics of L_P
  - Target: A (C) Horn Clause program capturing $[\![P]\!]$ (or, possibly, $[\![P]\!]^{\alpha}$)
- Block-based CFG. Each block represented as a *Horn clause*.
- Used for all analyses: aliasing, CHA/shape/types, data sizes, resources, etc.
- Allows supporting multiple languages.

Xcore Example: Control Flow Graph (CFG)

```
<fact>:
0x01: entsp (u6)    0x2
0x02: stw (ru6)     r0, sp[0x1]
0x03: ldw (ru6)     r1, sp[0x1]
0x04: ldc (ru6)     r0, 0x0
0x05: lss (3r)      r0, r0, r1
0x06: bf (ru6)      r0, 0x1 <0x08>
0x07: bu (u6)       0x2 <0x10>
0x08: mkmsk (rus)   r0, 0x1
0x09: retsp (u6)    0x2
0x10: ldw (ru6)     r0, sp[0x1]
0x11: sub (2rus)    r0, r0, 0x1
0x12: bl (u10)      -0xc <fact>
0x13: ldw (ru6)     r1, sp[0x1]
0x14: mul (l3r)     r0, r1, r0
0x15: retsp (u6)    0x2
```



Xcore Example: Block Representation

```
<fact>
0x01: entsp (u6)    0x2
0x02: stw (ru6)     r0, sp[0x1]
0x03: ldw (ru6)     r1, sp[0x1]
0x04: ldc (ru6)     r0, 0x0
0x05: lss (3r)      r0, r0, r1
0x06: bf (ru6)      r0, 0x1 <0x08>

0x07: bu (u6)       0x2 <0x10>
0x10: ldw (ru6)     r0, sp[0x1]
0x11: sub (2rus)    r0, r0, 0x1
0x12: bl (u10)      -0xc <fact>
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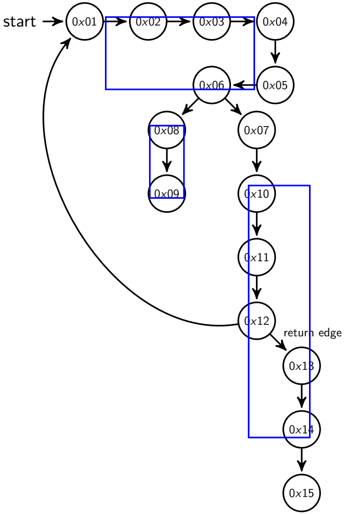
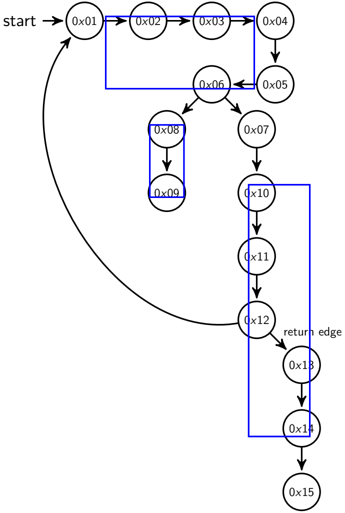


Xcore Example: Constrained Horn Clauses IR

```
:- entry fact/2.
fact(R0,R0_3):-
  entsp(_),
  stw(R0,Sp0x1),
  ldw(R1,Sp0x1),
  ldc(R0_1,0x0),
  lss(R0_2,R0_1,R1),
  bf(R0_2,_),
  bf01(R0_2,Sp0x1,R0_3,R1_1).

bf01(1,Sp0x1,R0_4,R1):-
  bu(_),
  ldw(R0_1,Sp0x1),
  sub(R0_2,R0_1,0x1),
  bl(_),
  fact(R0_2,R0_3),
  ldw(R1,Sp0x1),
  mul(R0_4,R1,R0_3),
  retsp(_).

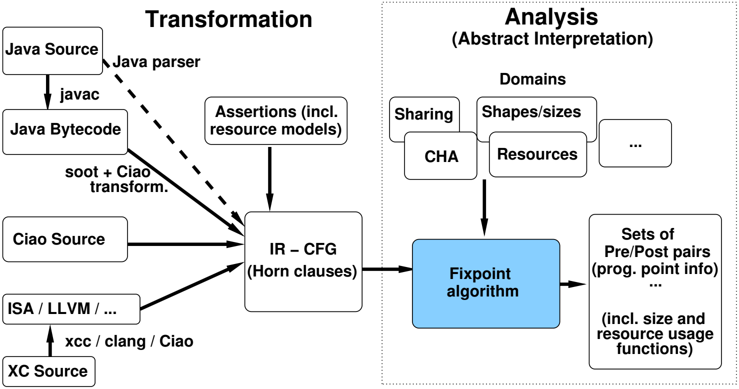
bf01(0,Sp0x1,R0,R1):-
  mkmsk(R0,0x1),
  retsp(_).
```



Generating the Intermediate Representation

- Typical tasks:
  - Generation of block-based CFG.
  - SSA transformation (e.g., splitting of input/output param).
  - Conversion of loops into recursions among blocks.
  - Branching, cases, dynamic dispatch → blocks w/same signature.
- Some specifics for Java:
  - Control flow graph is constructed from bytecode.
  - Elimination of stack variables.
  - Conversion to three-address statements.
  - Explicit representation of this and ret as extra block parameters.
- Some specifics for XC:
  - Control flow graph is constructed from ISA or LLVM IR representation.
  - Inferring block parameters.
  - Resolving branching to predicates with multiple clauses.
- Can be done via **partial evaluation of an interpreter** (implementing the semantics of the low-level code) w.r.t. the concrete low-level program or **directly** (cf. Futamura projections).

Analysis: CiaoPP Parametric AI Framework



- Analysis *parametric* w.r.t. abstractions, resources, ... (and languages).
- Efficient fixpoint algorithm for (C)HC IR.

[MH92, MGH94, BGH99, PH96, HPMS00, NMLH07]

[MH89, MH91, DLGH97, VB02, BLGH04, LGBH05, NBH06, MSHK07]

[MLH08, MKSH08, MMLH^{+}08, MHKS08, MKH09, LGBH10, MLLH08]

Efficient, Parametric Fixpoint Algorithm

- *Generic framework* for implementing HC-based analyses: given P (as a set of HCs) and abstract domain(s), computes $\mathrm{lfp}(S_P^\alpha) = [\![P]\!]_\alpha$, s.t. $[\![P]\!]_\alpha$ safely approximates $[\![P]\!]$.

→ Essentially efficient, incremental, abstract OLDT resolution of HC's.

- It maintains and computes as a result (simplified):
  - **A call-answer table**: with (multiple) entries $\{block : \lambda_{in} \mapsto \lambda_{out}\}$.
    - Exit states for calls to *block* satisfying precond λ_{in} meet postcond λ_{out}.
  - **A dependency arc table**: $\{A : \lambda_{inA} \Rightarrow B : \lambda_{inB}\}$.
    - Answers for call $A : \lambda_{inA}$ depend on the answers for $B : \lambda_{inB}$: (if exit for $B : \lambda_{inB}$ changes, exit for $A : \lambda_{inA}$ possibly also changes).
    - $Dep(B : \lambda_{inB})$ = the set of entries depending on $B : \lambda_{inB}$.
- Characteristics:
  - **Precision:** context-sensitivity / multivariance, prog. point info, ...
  - **Efficiency:** memoization, dependency tracking, SCCs, base cases, ...
  - **Genericity:** abstract domains are plugins, configurable, widening, ...
  - Handles mutually recursive methods.
  - Modular and *incremental*.
  - Handles library calls, externals, ...

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[MH89, MH92, MGH94, PH96, HPMS00, NMLH07]

Bottom-up vs. top-down

- Only bottom-up information (success):

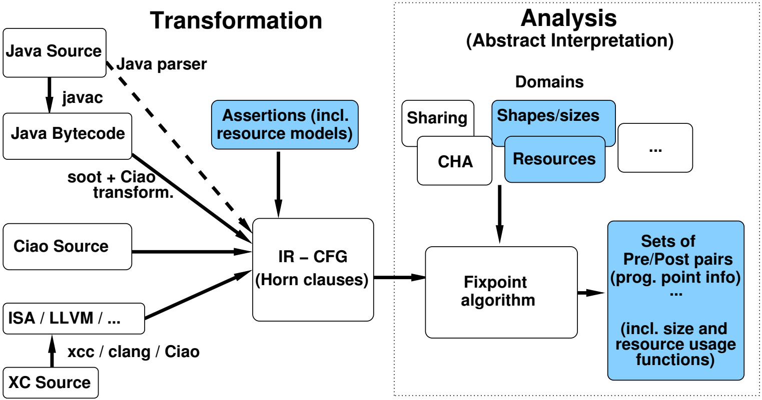
```
:- true pred sort(X,Y) => list(X), list(Y),
:- true pred sort(X,Y) => sorted(Y), perm(Y,X).
```

- With call information (top-down):

```
:- true pred sort(X,Y) :  list(X), var(Y)
                       => list(Y), sorted(Y), perm(Y,X)
                       +  resource(steps, len(X)**2),
                           no_fail.

:- true pred sort(X,Y) :  ground(X)
                       => indep(Y,X).
```

CiaoPP Resource Analysis



[DLH90, LGHD94, LGHD96, DLGHL94, DLGHL97, NMLGH07, MLNH07, MLGCH08, NMLH08]

[NMLH09, LGDB10, SLBH13, LKSGL13, SLH14] [LGK^{+}16, LBLGH16] [LGKLH16, HLGL^{+}16]

CiaoPP Resource Analysis

- The objective of the resource analysis is to obtain for each predicate/block *call - resource usage function pairs*:
  - Arithmetic functions providing lower/upper bounds on the resource usage of the predicate/block given the sizes of its input data for a particular entry condition.

Example

```
#pragma true nrev(x) :  list(x) ==>
    ( 0 <= resource(energy) && resource(energy) <= 1+length(x)**2) )
```

- **x** points to a list → energy consumed $\leq 1 + length(x)^2$

1. Programmer defines the resource consumption for basic elements (e.g., instructions, bytecodes, libraries, ...) – the "cost model."
2. System infers resource usage bound functions for rest of program. (Can be polynomial, exponential, logarithmic, ...)

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Some Examples of Resource Functions Inferred

Program	Resource	Usage Function	Metrics	Time
`client`	"bits received"	$\lambda x.8 \cdot x$	length	186
`color_map`	"unifications"	39066	size	176
`copy_files`	"files left open"	$\lambda x.x$	length	180
`eight_queen`	"queens movements"	19173961	length	304
`eval_polynom`	"FPU usage"	$\lambda x.2.5x$	length	44
`fib`	"arith. operations"	$\lambda x.2.17 \cdot 1.61^x +$ $0.82 \cdot (-0.61)^x - 3$	value	116
`grammar`	"phrases"	24	length/size	227
`hanoi`	"disk movements"	$\lambda x.2^x - 1$	value	100
`insert_stores`	"accesses Stores" "insertions Stores"	$\lambda n, m.n + k$ $\lambda n, m.n$	length	292
`perm`	"bytecode instructions"	$\lambda x.(\sum_{i=1}^{x} 18 \cdot x!)+$ $(\sum_{i=1}^{x} 14 \cdot \frac{x!}{i}) + 4 \cdot x!$	length	98
`power_set`	"output elements"	$\lambda x.\frac{1}{2} \cdot 2^{x+1}$	length	119
`qsort`	"lists parallelized"	$\lambda x.4 \cdot 2^x - 2x - 4$	length	144
`send_files`	"bytes read"	$\lambda x, y.x \cdot y$	length/size	179
`subst_exp`	"replacements"	$\lambda x, y.2xy + 2y$	size/length	153
`zebra`	"steps"	30232844295713061	size	292

- Different complexity functions, resources, size metrics, types of loops/recursion, etc.

Overview of the Analysis ("classical" view)

1. Supporting analyses (examples):
    - Sized types/shapes for size metrics (heap manipulating programs), and to simplify CFG and improve precision (class hierarchy analysis).
    - Sharing analysis for correctness (conservative: only when there is no sharing among data structures).
    - *Non-failure* (no exceptions) inferred for non-trivial lower bounds.
    - *Determinacy* (mutual exclusion) to obtain tighter bounds.
2. Set up recurrence equations representing the size of each (relevant) output argument as a function of the input data sizes.
    - Size metrics are derived from inferred shape (type) information.
    - Data dependency graphs determine *relative* sizes of variable contents.
3. Compute bounds to the solutions of these recurrence equations to obtain output argument sizes as functions of input sizes.
    - Using internal recurrence solver, or the interfaces with Mathematica, Parma, PUBS, Matlab, etc.

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4. E.g.:

```
#pragma true conc(x,y) : list * list
      ==> ( length(x)+length(y) <= size(ret) &&
            size(ret) <= length(x)+length(y) )
```

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5. Use the size information to set up recurrence equations representing the computational cost of each block and compute bounds to their solutions to obtain **resource usage functions**.

Resource Analysis as an Abstract Interpretation

[SLH14, SLBH13]

- In classical CiaoPP resource analysis the last steps (setting up and solving recurrences) were not implemented as an abstract domain.
- We have recently integrated resource analysis as an *abstract domain "plug-in"* of the generic analysis fixpoint –we get for free:
  - Multivariance: e.g., separate different call patterns for same block:

```
sort(lst(int),var) ...  sort(lst(flt),var) ...  sort(var,lst(int))
```

  - Easier combination with other domains.
  - Easier integration w/static debugging/verification and rt-checking.
  - Many other engineering advantages.
- New domain for size analysis (*sized types*) that infers bounds on the size of data structures *and substructures*.

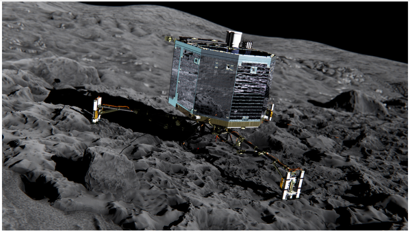
listnum	*sized*(*listnum*)
`listnum -> []` `listnum -> [num | listnum]`	$listnum^{(\alpha,\beta)}\left(num^{(\gamma,\delta)}_{\langle \cdot,1\rangle}\right)$

- Competitive results with state-of-the-art systems (e.g., RAML).
- Used in the XC energy analysis.

Energy Consumption Analysis – Motivation

Energy consumption of computing technologies is a major concern

From high-perf. computing and cloud servers to mobile phones, wearables, implantable/portable medical devices, micro-spacecraft, sensors ...



Energy Consumption Analysis – Approach

Requires low-level models – approach: [NMLH08]

- Specialize generic resource analysis with instruction-level models:
  - Provide energy and data size assertions for each individual instruction. (Energy and data sizes can be constants or *functions*.)
- CiaoPP then generates statically safe upper- and lower-bound energy consumption functions.

- Initially applied to Java bytecode: [NMLH08]
  - Java bytecode energy consumption models available for simple processors –upper bound consumption per bytecode in joules:

Opcode	Inst. Cost in μJ	Mem. Cost in μJ	Total Cost in in μJ
`iadd`	.957860	2.273580	3.23144
`isub`	.957360	2.273580	3.230.94
...	...	...	...

  - Encouraging results: meaningful functions inferred in many cases.
  - But no comparison with actual device consumption.

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⇒ Addressed recently: [LKSGL13, LGK+16, LBLGH16]

- Analysis of (embedded) programs written in XC, on XMOS processors.
- Using more sophisticated *ISA-level energy models* for XMOS XS1, developed by Bristol & XMOS.
- Comparing to measured energy consumption.

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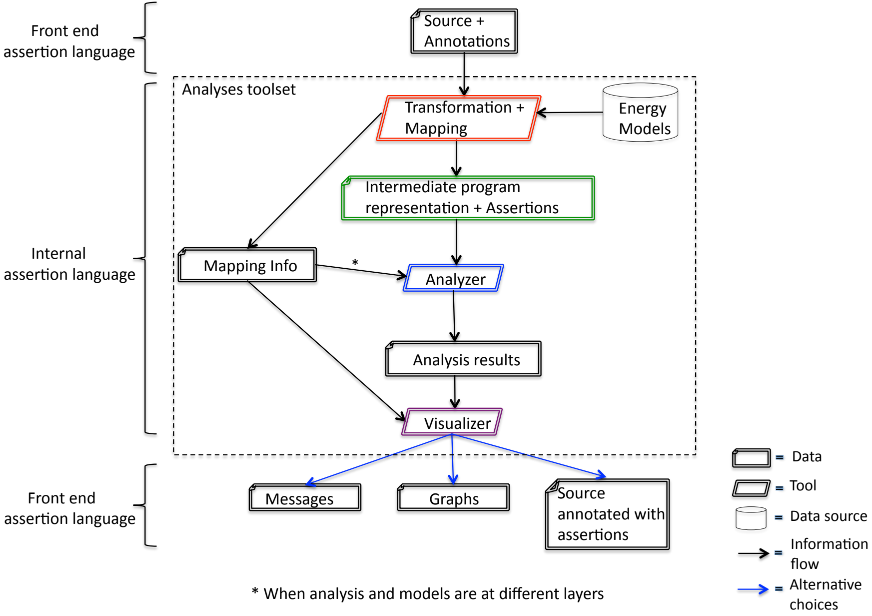
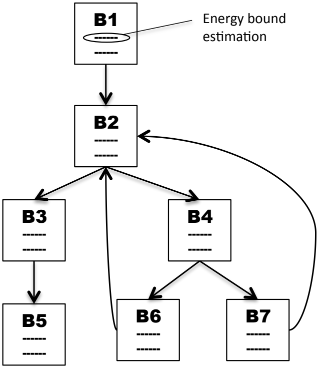


* When analysis and models are at different layers

Modeling at the Instruction Level

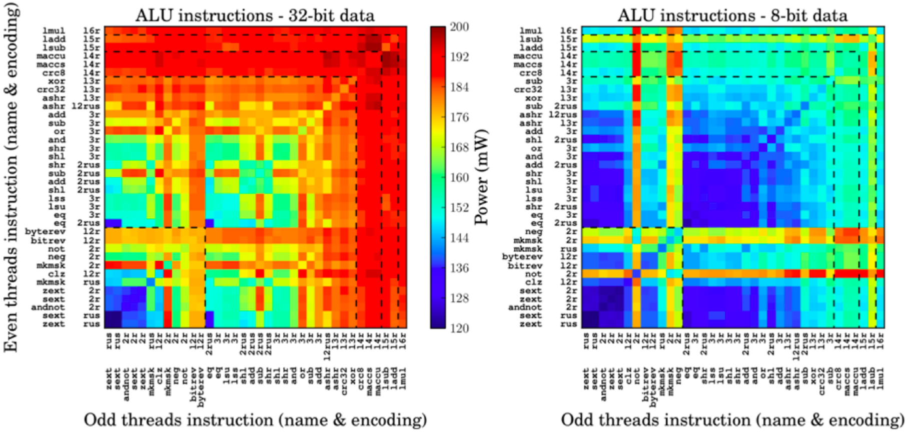


- Each instruction is profiled (using, e.g., an Evolutionary Algorithm – EA) to derive upper- and lower-bound energy estimates.
- These are combined using static analysis.

+ Very compositional.

Low-level ISA characterization – interference

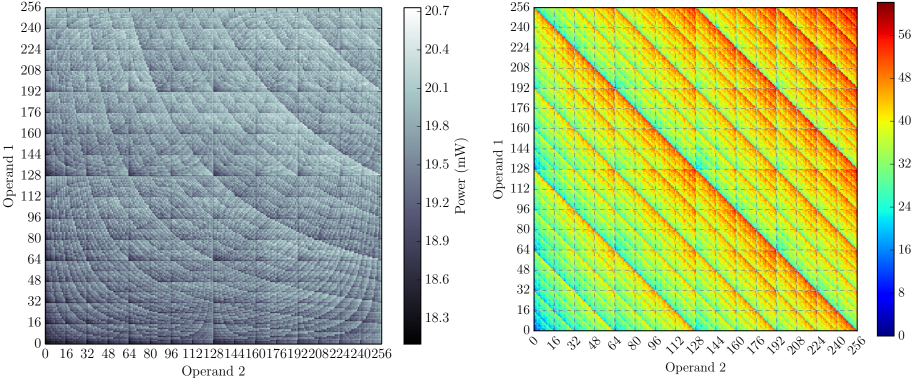
Obtaining the cost model: energy consumption/instruction; interference.



Eder, Kerrison – Bristol U / XMOS.

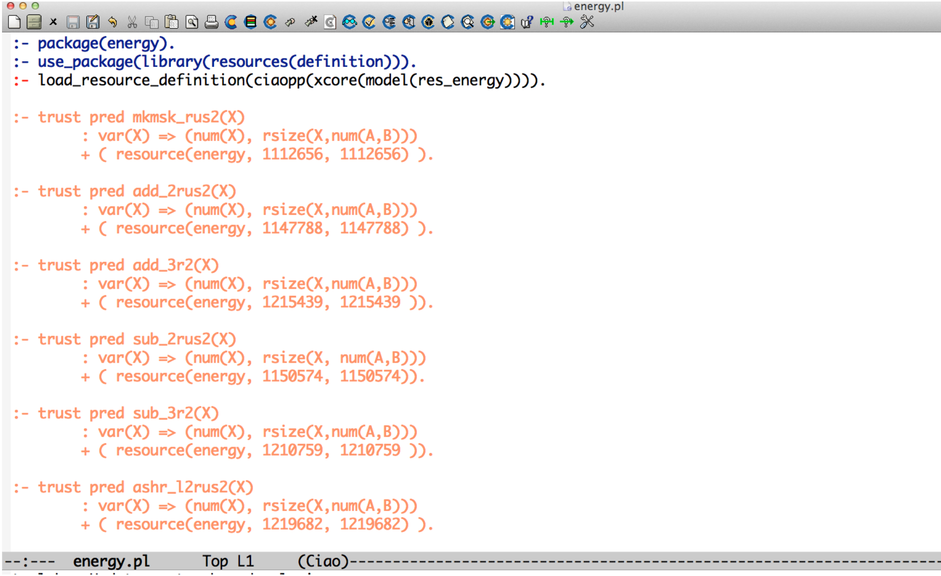
Low-level ISA characterization – operand size

Obtaining the cost model: energy consumption/instruction; operand size.



Eder, Kerrison – Bristol U / XMOS.

Energy model, expressed in the Ciao assertion language



```
:- package(energy).
:- use_package(library(resources(definition))).
:- load_resource_definition(ciaopp(xcore(model(res_energy)))).

:- trust pred mkmsk_rus2(X)
        : var(X) => (num(X), rsize(X,num(A,B)))
        + ( resource(energy, 1112656, 1112656) ).

:- trust pred add_2rus2(X)
        : var(X) => (num(X), rsize(X,num(A,B)))
        + ( resource(energy, 1147788, 1147788) ).

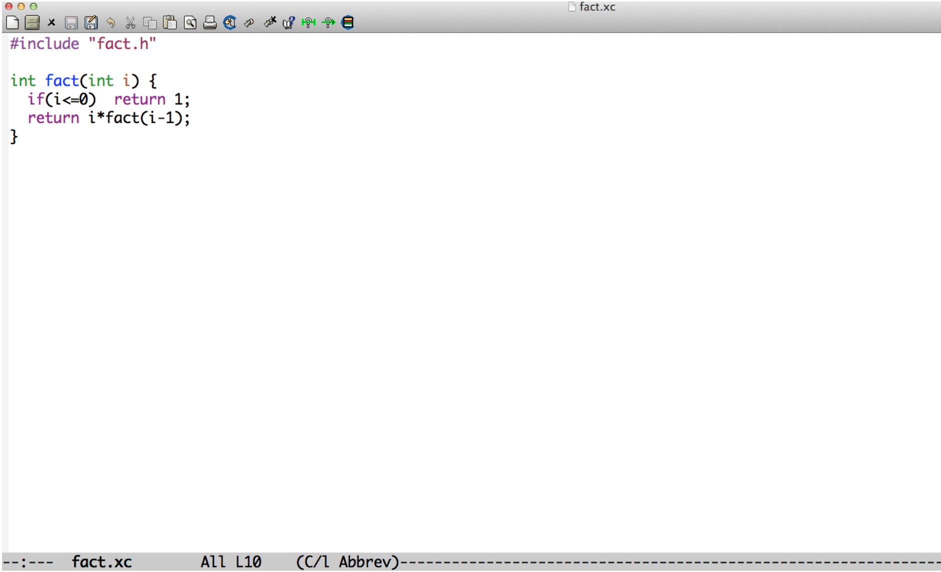
:- trust pred add_3r2(X)
        : var(X) => (num(X), rsize(X,num(A,B)))
        + ( resource(energy, 1215439, 1215439 )).

:- trust pred sub_2rus2(X)
        : var(X) => (num(X), rsize(X, num(A,B)))
        + ( resource(energy, 1150574, 1150574)).

:- trust pred sub_3r2(X)
        : var(X) => (num(X), rsize(X,num(A,B)))
        + ( resource(energy, 1210759, 1210759 )).

:- trust pred ashr_l2rus2(X)
        : var(X) => (num(X), rsize(X,num(A,B)))
        + ( resource(energy, 1219682, 1219682) ).
```

XC Source

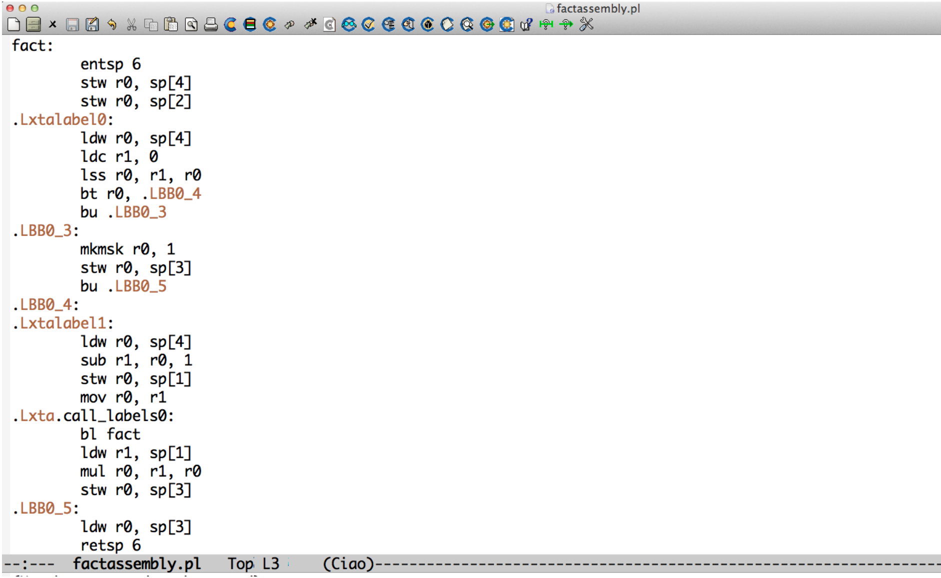


```
#include "fact.h"

int fact(int i) {
  if(i<=0)  return 1;
  return i*fact(i-1);
}
```

```
--:---  fact.xc        All L10    (C/l Abbrev)------------------------------------------------------
```

Assembly Code



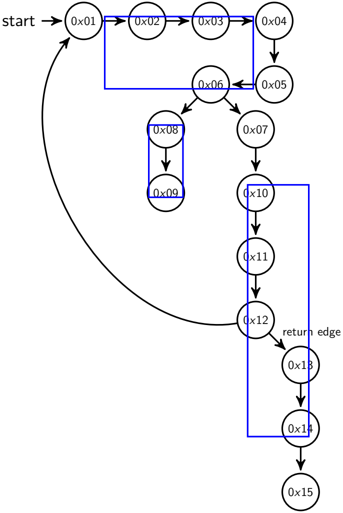
```
fact:
        entsp 6
        stw r0, sp[4]
        stw r0, sp[2]
.Lxtalabel0:
        ldw r0, sp[4]
        ldc r1, 0
        lss r0, r1, r0
        bt r0, .LBB0_4
        bu .LBB0_3
.LBB0_3:
        mkmsk r0, 1
        stw r0, sp[3]
        bu .LBB0_5
.LBB0_4:
.Lxtalabel1:
        ldw r0, sp[4]
        sub r1, r0, 1
        stw r0, sp[1]
        mov r0, r1
.Lxta.call_labels0:
        bl fact
        ldw r1, sp[1]
        mul r0, r1, r0
        stw r0, sp[3]
.LBB0_5:
        ldw r0, sp[3]
        retsp 6
--:---  factassembly.pl   Top L3    (Ciao)------------------------------------------------
```

Xcore Example: Block Representation

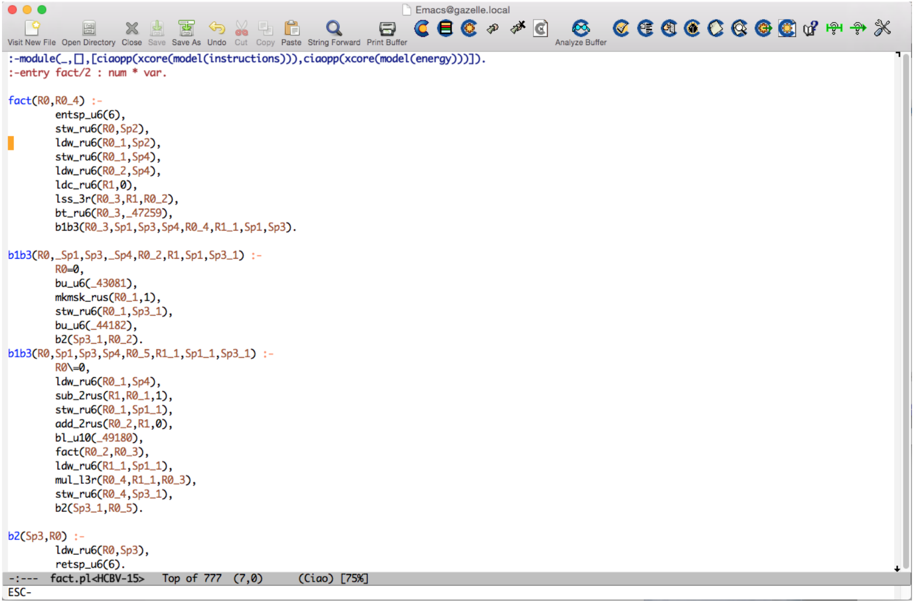
```
<fact>
0x01: entsp (u6)    0x2
0x02: stw (ru6)     r0, sp[0x1]
0x03: ldw (ru6)     r1, sp[0x1]
0x04: ldc (ru6)     r0, 0x0
0x05: lss (3r)      r0, r0, r1
0x06: bf (ru6)      r0, 0x1 <0x08>

0x07: bu (u6)       0x2 <0x10>
0x10: ldw (ru6)     r0, sp[0x1]
0x11: sub (2rus)    r0, r0, 0x1
0x12: bl (u10)      -0xc <fact>
0x13: ldw (ru6)     r1, sp[0x1]
0x14: mul (l3r)     r0, r1, r0
0x15: retsp (u6)    0x2

0x08: mkmsk (rus)   r0, 0x1
0x09: retsp (u6)    0x2
```



Horn Clause Representation



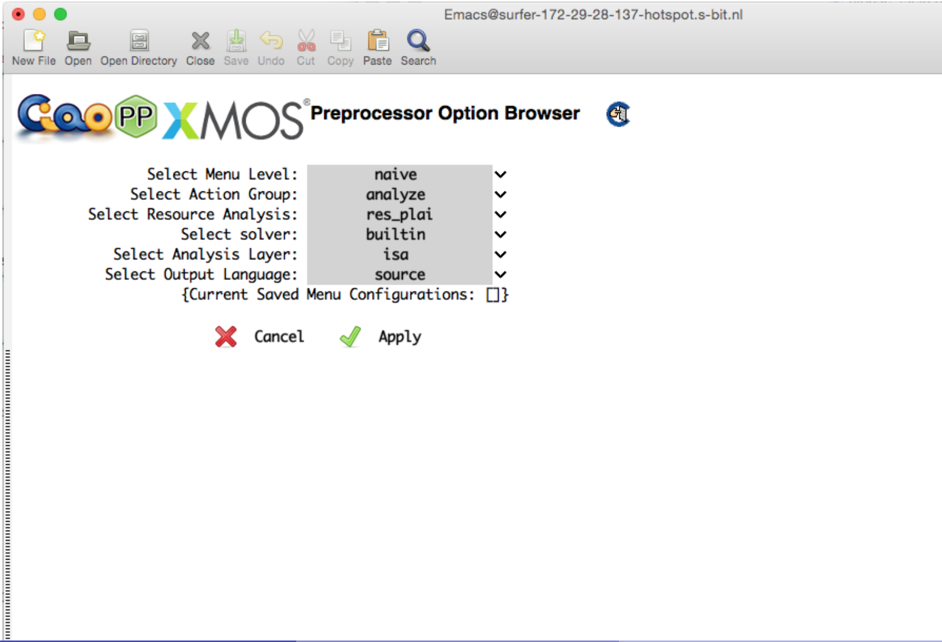
```
:-module(_,[],[ciaopp(xcore(model(instructions))),ciaopp(xcore(model(energy)))]).
:-entry fact/2 : num * var.

fact(R0,R0_4) :-
        entsp_u6(6),
        stw_ru6(R0,Sp2),
        ldw_ru6(R0_1,Sp2),
        stw_ru6(R0_1,Sp4),
        ldw_ru6(R0_2,Sp4),
        ldc_ru6(R1,0),
        lss_3r(R0_3,R1,R0_2),
        bt_ru6(R0_3,_47259),
        b1b3(R0_3,Sp1,Sp3,Sp4,R0_4,R1_1,Sp1,Sp3).

b1b3(R0,_Sp1,Sp3,_Sp4,R0_2,R1,Sp1,Sp3_1) :-
        R0=0,
        bu_u6(_43081),
        mkmsk_rus(R0_1,1),
        stw_ru6(R0_1,Sp3_1),
        bu_u6(_44182),
        b2(Sp3_1,R0_2).
b1b3(R0,Sp1,Sp3,Sp4,R0_5,R1_1,Sp1_1,Sp3_1) :-
        R0\=0,
        ldw_ru6(R0_1,Sp4),
        sub_2rus(R1,R0_1,1),
        stw_ru6(R0_1,Sp1_1),
        add_2rus(R0_2,R1,0),
        bl_u10(_49180),
        fact(R0_2,R0_3),
        ldw_ru6(R1_1,Sp1_1),
        mul_l3r(R0_4,R1_1,R0_3),
        stw_ru6(R0_4,Sp3_1),
        b2(Sp3_1,R0_5).

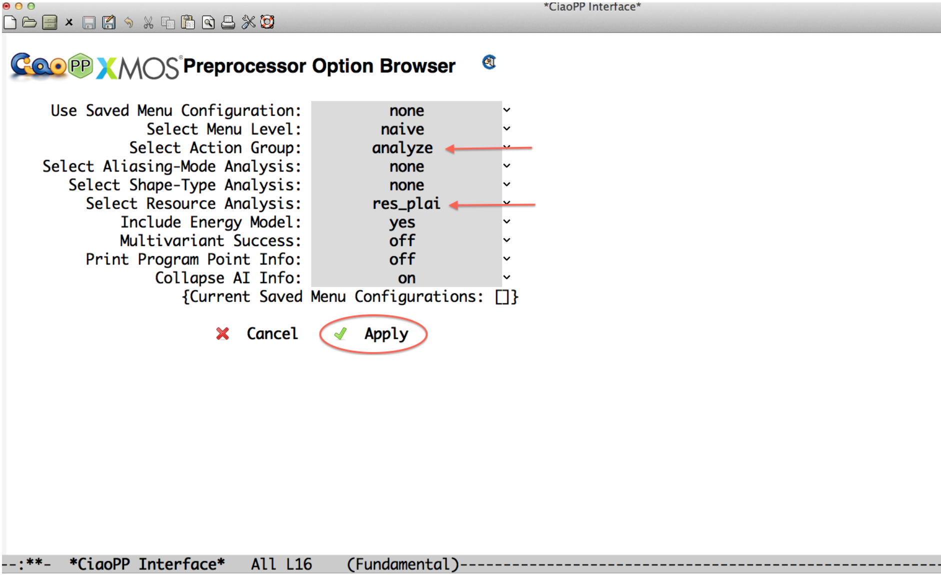
b2(Sp3,R0) :-
        ldw_ru6(R0,Sp3),
        retsp_u6(6).
-:---  fact.pl<HCBV-15>   Top of 777  (7,0)      (Ciao) [75%]
ESC-
```

CiaoPP Menu



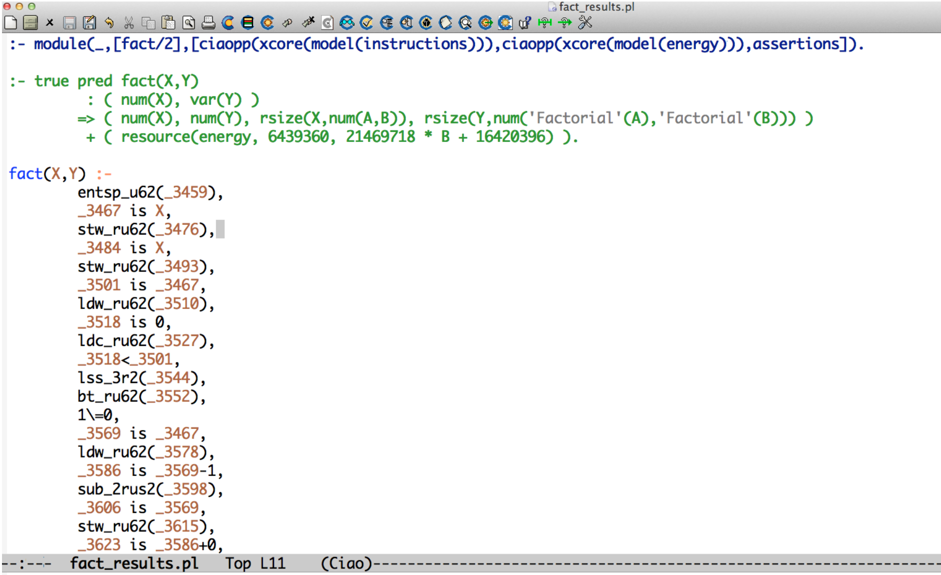
Select Menu Level: naive
Select Action Group: analyze
Select Resource Analysis: res_plai
Select solver: builtin
Select Analysis Layer: isa
Select Output Language: source
{Current Saved Menu Configurations: []}

Select Resource Analysis



```
    Use Saved Menu Configuration:   none
               Select Menu Level:   naive
             Select Action Group:   analyze
  Select Aliasing-Mode Analysis:   none
      Select Shape-Type Analysis:   none
        Select Resource Analysis:   res_plai
            Include Energy Model:   yes
           Multivariant Success:   off
          Print Program Point Info:   off
               Collapse AI Info:   on
         {Current Saved Menu Configurations: []}
```

Analysis Results

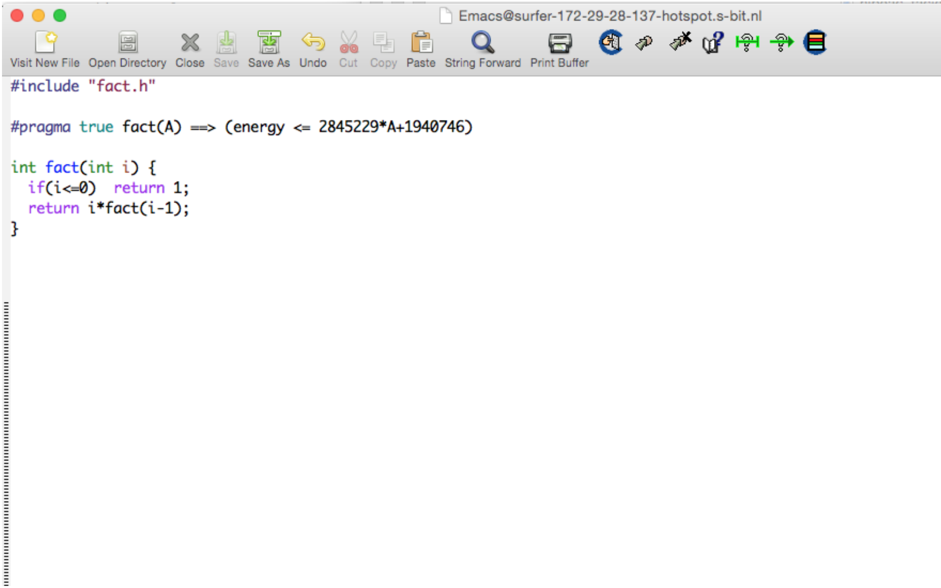


```
:- module(_,[fact/2],[ciaopp(xcore(model(instructions))),ciaopp(xcore(model(energy))),assertions]).

:- true pred fact(X,Y)
         : ( num(X), var(Y) )
        => ( num(X), num(Y), rsize(X,num(A,B)), rsize(Y,num('Factorial'(A),'Factorial'(B))) )
         + ( resource(energy, 6439360, 21469718 * B + 16420396) ).

fact(X,Y) :-
        entsp_u62(_3459),
        _3467 is X,
        stw_ru62(_3476),
        _3484 is X,
        stw_ru62(_3493),
        _3501 is _3467,
        ldw_ru62(_3510),
        _3518 is 0,
        ldc_ru62(_3527),
        _3518<_3501,
        lss_3r2(_3544),
        bt_ru62(_3552),
        1\=0,
        _3569 is _3467,
        ldw_ru62(_3578),
        _3586 is _3569-1,
        sub_2rus2(_3598),
        _3606 is _3569,
        stw_ru62(_3615),
        _3623 is _3586+0,
```

Analysis Output

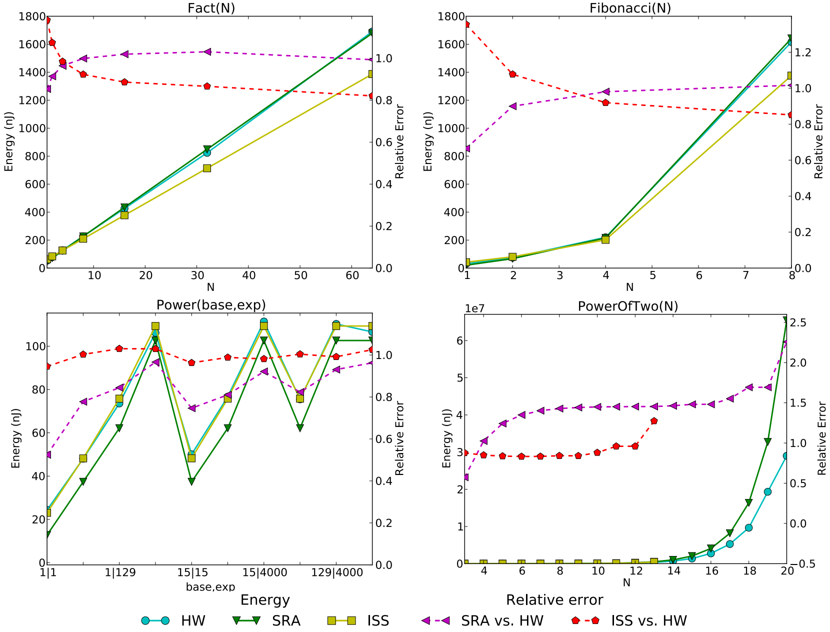


```
#include "fact.h"

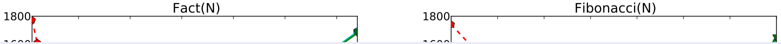
#pragma true fact(A) ==> (energy <= 2845229*A+1940746)

int fact(int i) {
  if(i<=0)  return 1;
  return i*fact(i-1);
}
```

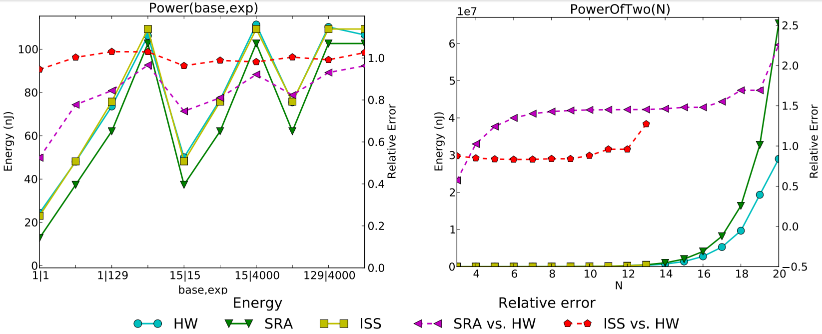
Some Results [LKSGL13]



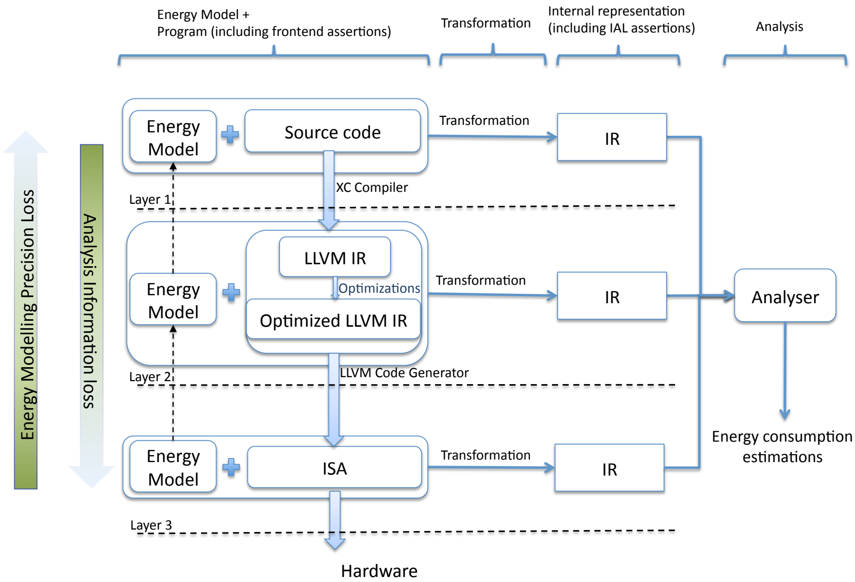
Some Results [LKSGL13]



- SRA provides results *beyond what is possible with simulation* (as test run-time increases, ISS becomes impractically long).
- SRA showed already promising accuracy in comparison with ISS and the HW (but still relatively simple benchmarks).
- Simulation time limits the usefulness of ISS method, whereas equation solving limits SRA.



IR Level Trade-offs



XC Analysis Results (FIR Filter, LLVM IR level)

```
@#pragma true fir(xn, coeffs, state, N) :
             (3347178*N + 13967829 <= energy &&
              energy <= 3347178*N + 14417829)
```

```
int fir(int xn, int coeffs[], int state[], int ELEMENTS)
{
  unsigned int ynl; int ynh;
  ynl = (1<<23); ynh = 0;
  for(int j=ELEMENTS-1; j!=0; j--) {
      state[j] = state[j-1];
      {ynh, ynl} = macs(coeffs[j], state[j], ynh, ynl);
  }
  state[0] = xn;
  {ynh, ynl} = macs(coeffs[0], xn, ynh, ynl);
  if (sext(ynh,24) == ynh) {
      ynh = (ynh << 8) | (((unsigned) ynl) >> 24);}
  else if (ynh < 0) { ynh = 0x80000000; }
  else { ynh = 0x7fffffff; }
  return ynh;
}
```

XC Analysis Results (FIR Filter, LLVM IR level)

```
@#pragma true fir(xn, coeffs, state, N) :
              (3347178*N + 13967829 <= energy &&
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{
  unsigned int ynl; int ynh;
  ynl = (1<<23); ynh = 0;
  for(int j=ELEMENTS-1; j!=0; j--) {
      state[j] = state[j-1];
      {ynh, ynl} = macs(coeffs[j], state[j], ynh, ynl);
  }
  state[0] = xn;
  {ynh, ynl} = macs(coeffs[0], xn, ynh, ynl);
  if (sext(ynh,24) == ynh) {
      ynh = (ynh << 8) | (((unsigned) ynl) >> 24);}
  else if (ynh < 0) { ynh = 0x80000000; }
  else { ynh = 0x7fffffff; }
  return ynh;
}
```

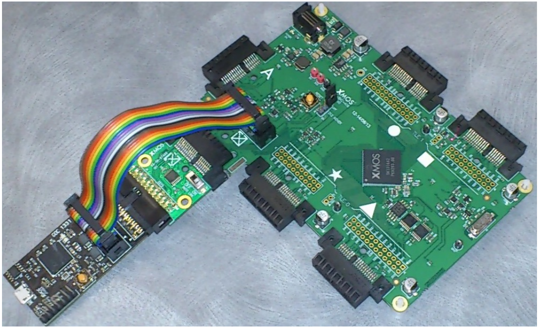
XC Energy Analysis Results – LLVMIR level

[LGKL+15]

Program	**Analysis at LLVMIR level** **Energy consumption funtions (in NJ)**
`fact(N)`	$28.4\ N + 22.4$
`fibonacci(N)`	$37.53 + 42.3 \times 1.62^N + 11.68 \times (-0.62)^N$
`sqr(N)`	$10.52\ N^2 + 55.79\ N + 16.5$
`power_of_two(N)`	$49.2 \times 2^N - 31.5$
`reverse(N,M)`	$20.50\ N + 72.98$
`concat(N,M)`	$69.14\ N + 69.14\ M + 14.12$
`mat_mult(N,M)`	$44.71\ N^3 + 72.47\ N^2 + 52.52\ N + 25.49$
`sum_facts(N,M)`	$69.14\ N + 69.14\ M + 14.12$
`fir(N)`	$33.47\ N + 141.6$
`biquad(N)`	$165.3\ N + 54.45$

Measuring Power Consumption on the Hardware

- XMOS XTAG3 measurement circuit.
- Plugs into XMOS XS1 board.



We compare these HW measurements with:

- Static Resource Analysis (SRA).
- Instruction Set Simulation (ISS).

Accuracy vs. HW measurements (ISA and LLVMIR) [LGKL+15]

Program	**Error vs. HW**		**ISA/LLVMIR**
	isa	**llvmir**	
`fact(N)`	2.86%	4.50%	0.94
`fibonacci(N)`	5.41%	11.94%	0.92
`sqr(N)`	1.49%	9.31%	0.91
`power_of_two(N)`	4.26%	11.15%	0.93
`Average`	**3.50%**	**9.20%**	**0.92**
`reverse(N,M)`	N/A	2.18%	N/A
`concat(N,M)`	N/A	8.71%	N/A
`mat_mult(N,M)`	N/A	1.47%	N/A
`sum_facts(N,M)`	N/A	2.42%	N/A
`fir(N)`	N/A	0.63%	N/A
`biquad(N)`	N/A	2.34%	N/A
`Average`	**N/A**	**3.0%**	**N/A**
`Gobal Avg.`	**3.50%**	**5.48%**	**0.92**

Accuracy vs. HW measurements (ISA and LLVMIR) [LGKL+15]

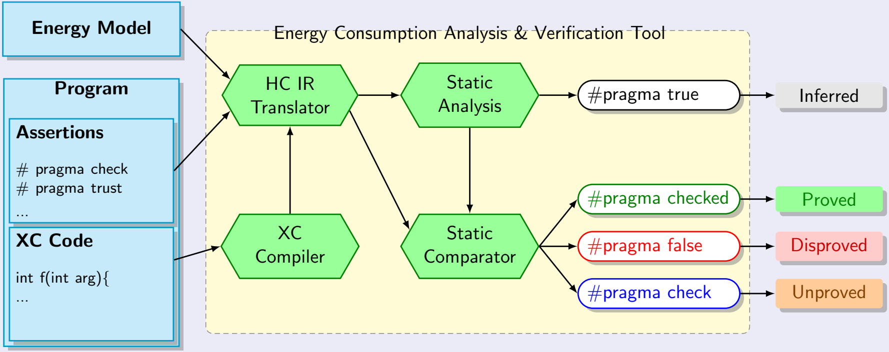
Program	**Error vs. HW**		**ISA/LLVMIR**
	isa	**llvmir**	
`fact(N)`	2.86%	4.50%	0.94
`fibonacci(N)`	5.41%	11.94%	0.92
`sqr(N)`	1.49%	9.31%	0.91
`power_of_two(N)`	4.26%	11.15%	0.93
`Average`	**3.50%**	**9.20%**	**0.92**
`Average`	**N/A**	**3.0%**	**N/A**
`Gobal Avg.`	**3.50%**	**5.48%**	**0.92**

- ISA analysis estimations are reasonably accurate.
- ISA estimations are more accurate than LLVM estimations.
- LLVM estimations are close to ISA estimations.
- Some programs cannot be analysed at the ISA level but can be analyzed at the LLVM level.

Applications

Performance debugging and verification, resource-oriented optimization, heterogeneous computers, QoS, . . .

XC Energy Consumption Verification Tool (based on CiaoPP)



[LHKLH15]

XC Program (FIR Filter) w/Energy Specification [LHKLH15]

```
#pragma check fir(xn, coeffs, state, N) :
          (1 <= N) ==> (energy <= 416079189)

#pragma true fir(xn, coeffs, state, N) :
              (3347178*N + 13967829 <= energy &&
               energy <= 3347178*N + 14417829)

#pragma checked fir(xn, coeffs, state, N) :
              (1 <= N && N <= 120) ==> (energy <= 416079189)

#pragma false fir(xn, coeffs, state, N) :
              (121 <= N) ==> (energy <= 416079189)

int fir(int xn, int coeffs[], int state[], int ELEMENTS)
{
  unsigned int ynl; int ynh;
  ynl = (1<<23); ynh = 0;
  for(int j=ELEMENTS-1; j!=0; j--) {
      state[j] = state[j-1];
      {ynh, ynl} = macs(coeffs[j], state[j], ynh, ynl);
  }
  state[0] = xn;
  {ynh, ynl} = macs(coeffs[0], xn, ynh, ynl);
  if (sext(ynh,24) == ynh) {
      ynh = (ynh << 8) | (((unsigned) ynl) >> 24);}
```

XC Program (FIR Filter) w/Energy Specification [LHKLH15]

```
#pragma check fir(xn, coeffs, state, N) :
          (1 <= N) ==> (energy <= 416079189)

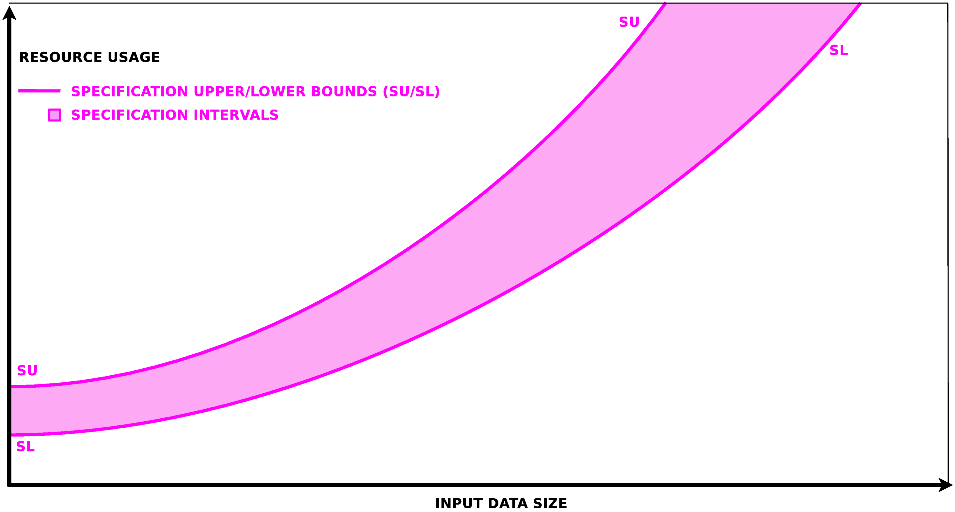
#pragma true fir(xn, coeffs, state, N) :
             (3347178*N + 13967829 <= energy &&
              energy <= 3347178*N + 14417829)

#pragma checked fir(xn, coeffs, state, N) :
             (1 <= N && N <= 120) ==> (energy <= 416079189)

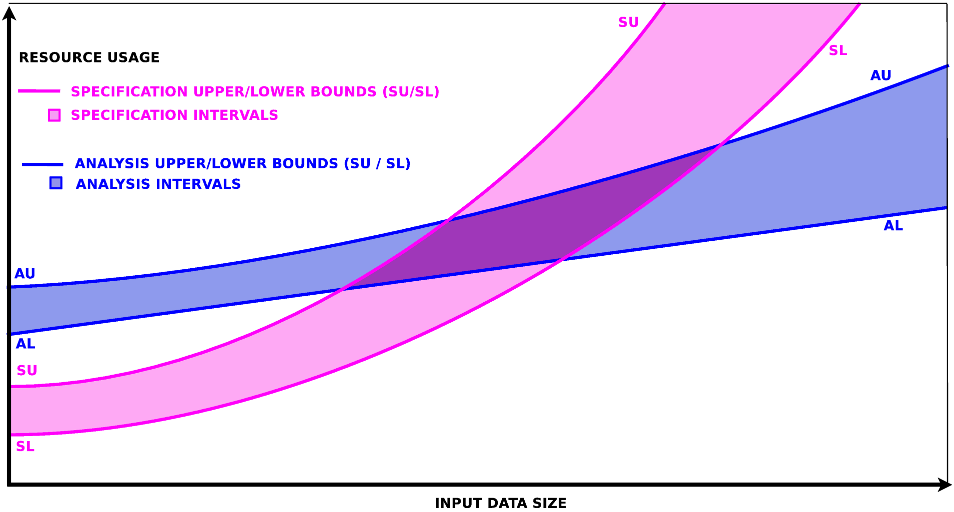
#pragma false fir(xn, coeffs, state, N) :
             (121 <= N) ==> (energy <= 416079189)

int fir(int xn, int coeffs[], int state[], int ELEMENTS)
{
  unsigned int ynl; int ynh;
  ynl = (1<<23); ynh = 0;
  for(int j=ELEMENTS-1; j!=0; j--) {
      state[j] = state[j-1];
      {ynh, ynl} = macs(coeffs[j], state[j], ynh, ynl);
  }
  state[0] = xn;
  {ynh, ynl} = macs(coeffs[0], xn, ynh, ynl);
  if (sext(ynh,24) == ynh) {
      ynh = (ynh << 8) | (((unsigned) ynl) >> 24);}
```

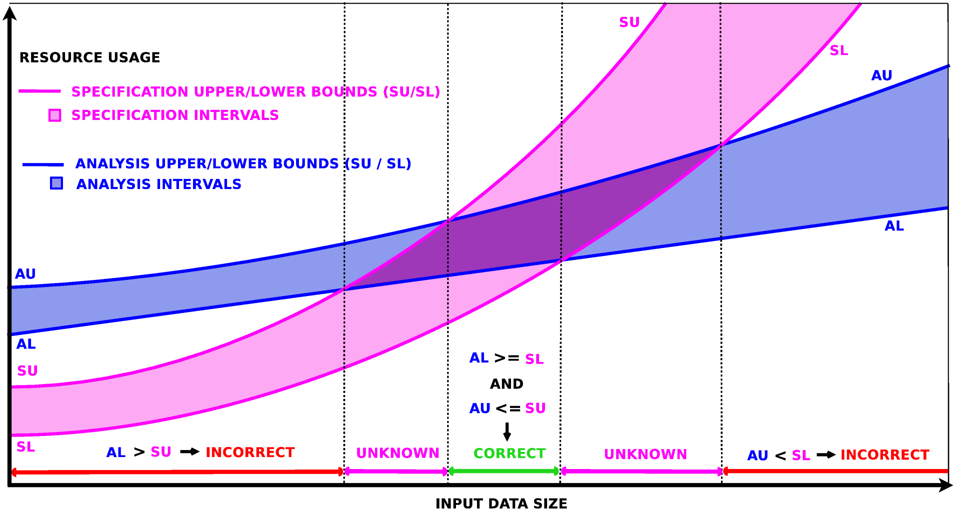
Resource Usage Verification – Function Comparisons



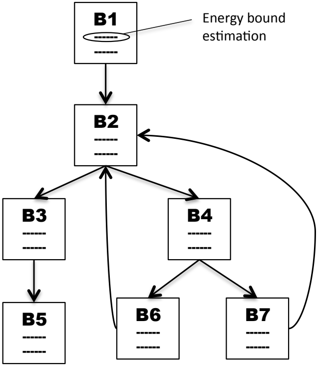
Resource Usage Verification – Function Comparisons



Resource Usage Verification – Function Comparisons



Modeling at the Instruction Level



- Each instruction is profiled (using, e.g., an Evolutionary Algorithm – EA) to derive upper- and lower-bound energy estimates.
- These are combined using static analysis.

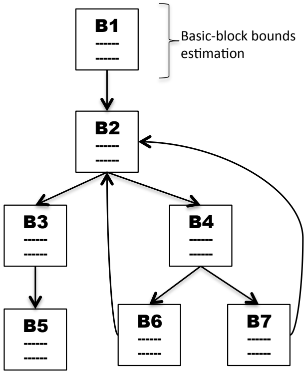
\+ Very compositional.

– Bounds obtained are *very conservative*.

– Dependence among instructions is not modeled (or complex).

Modeling at the Basic Block Level

[LBLGH16]



- Each basic block is profiled using the EA and upper/lower bounds estimated for each block.
- Bounds over basic blocks are composed (by static analysis) to infer the bounds over whole program.

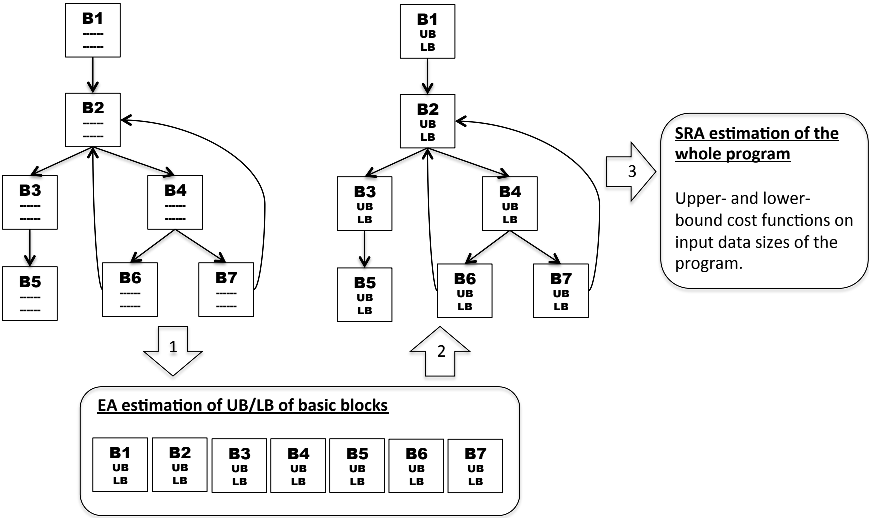
\+ Inter-instruction dependence is captured within the blocks: more precise bounds.

\+ The EA is precise and practical since no data dependent branching within a block.

\+ Infers functions of input data sizes.

– Inter-block dependence may be over- or under-estimated.

Overview of our Approach

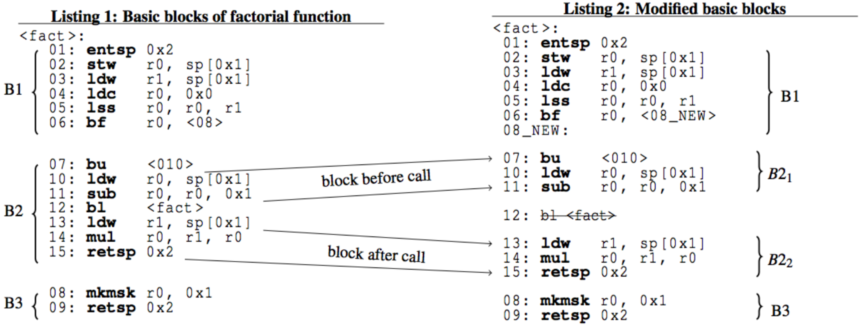


SRA estimation of the whole program

Upper- and lower-bound cost functions on input data sizes of the program.

EA estimation of UB/LB of basic blocks

Dividing the Program into Basic Blocks



Listing 1: Basic blocks of factorial function

```
<fact>:
     01: entsp 0x2
     02: stw   r0, sp[0x1]
     03: ldw   r1, sp[0x1]
B1   04: ldc   r0, 0x0
     05: lss   r0, r0, r1
     06: bf    r0, <08>

     07: bu    <010>
     10: ldw   r0, sp[0x1]
     11: sub   r0, r0, 0x1
B2   12: bl    <fact>
     13: ldw   r1, sp[0x1]
     14: mul   r0, r1, r0
     15: retsp 0x2

B3   08: mkmsk r0, 0x1
     09: retsp 0x2
```

Listing 2: Modified basic blocks

```
<fact>:
 01: entsp 0x2
 02: stw   r0, sp[0x1]
 03: ldw   r1, sp[0x1]
 04: ldc   r0, 0x0               B1
 05: lss   r0, r0, r1
 06: bf    r0, <08_NEW>
 08_NEW:

 07: bu    <010>
 10: ldw   r0, sp[0x1]           B2_1
 11: sub   r0, r0, 0x1

 12: ~~bl <fact>~~

 13: ldw   r1, sp[0x1]
 14: mul   r0, r1, r0            B2_2
 15: retsp 0x2

 08: mkmsk r0, 0x1               B3
 09: retsp 0x2
```

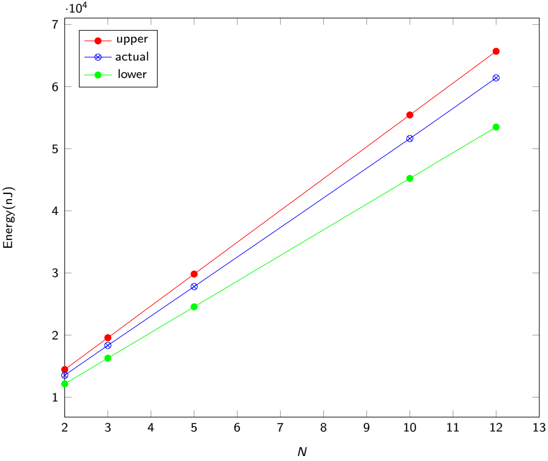
Experimental Evaluation (XMOS XS1 architecture)

Program	**Upper/Lower Bounds (nJ)**$\times 10^3$	**vs. HW**
$fact(N)$	$E_u = 5.1\ N + 4.2$ $E_l = 4.1\ N + 3.8$	7% -11.7%
$fibonacci(N)$	$E_u = 5.2\ lucas(N) + 6\ fib(N) - 6.6$ $E_l = 4.5\ lucas(N) + 5\ fib(N) - 4.2$	8.71% -4.69%
$reverse(N)$	$E_u = 3.7\ N + 13.3$ $E_l = 2.95\ N + 12$	8% -8.8%
$findMax(N)$	$E_u = 5\ N + 6.9$ $E_l = 3.3\ N + 5.6$	8.7% -9.1%
$fir(N)$	$E_u = 6\ N + 26.4$ $E_l = 4.8\ N + 22.9$	8.9% -9.7%
$biquad(N)$	$E_u = 29.6\ N + 10$ $E_l = 23.5\ N + 9$	9.8% -11.9%

- EA times vary depending upon the initialization parameters.
  - On average within 150-200 min.
- Static analysis times are relatively small $\approx 4sec$.

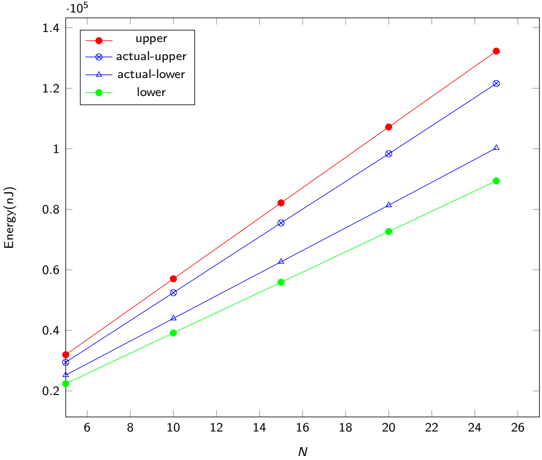
Experimental Results (Benchmark with no Data Dependent Branching)

factorial(x): 7% over- and 11% under-approximation for random runs with different inputs.



Experimental Results (Benchmark with Data Dependent Branching)

findMax(arr,N): 8.7% over- and 9% under-approximation from actual upper- and lower-bounds (ascending vs. descending sorted array).



Inferring Accumulated Cost

[LGKLH16, HLGL$^+$16]

- For verification purposes *safe* upper and lower bounds are required → standard/classical cost, but
- for optimization → accumulated cost:
  - Guides the program developer to identify parts that should be optimized, because of their greater impact on the total cost.
- The *standard cost* of a procedure q is

  → the cost of a *single call* to q, denoted $\mathcal{C}_q(\bar{n})$.
- The *accumulated cost* of a procedure q is defined in the context of a call to another (or the same) procedure p.
  - It is the *addition of the (accumulated) costs of all calls to* q originated during the computation of a (single) call to p.
  - Denoted $\mathcal{C}_p^q(\bar{n})$, *"the accumulated cost of q when called from p"*.

Example: Variance of an Array of Integers

Assume that both mean() and variance () are declared as cost centers.

Naive implementation: mean() is responsible for most of the cost of the call to variance ().

```
int  variance(int * Arr, int N){
  int tmp[N], i = N;
  while(i > 0) {
    i--;
    tmp[i] = (Arr[i] - mean(Arr, N));
    tmp[i] = tmp[i] * tmp[i];
    }
  return mean(tmp, N);
}
```

Standard Cost:

$$\mathcal{C}_{mean}(N') \in \mathcal{O}(N')$$

$$\mathcal{C}_{variance}(N) \in \mathcal{O}(N^2)$$

Accumulated Cost:

$$\mathcal{C}^{mean}_{variance}(N) \in \mathcal{O}(N^2)$$

$$\mathcal{C}^{variance}_{variance}(N) \in \mathcal{O}(N)$$

Obvious improvement: move mean(Arr, N) outside the loop.

```
int  variance(int * Arr, int N){
  int tmp[N], int i = N;
  int m = mean(Arr, N);
  while(i > 0) {
    i--;
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    tmp[i] = tmp[i] * tmp[i];
  }
  return mean(tmp, N);
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Our system infers that the costs accumulated in the variance () and mean() are linear.

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Accumulated Cost: Experimental Results

Cost-Centers & Input Sizes	Accumulated Cost UB	Static vs. Dyn	Standard Cost UB	#Calls
$variance(n)^*$	1	0%	$2n^2$	1
$sq\_diff(m_1, m_2)$	$n-1$	0%	$2m_1m_2 - 2m_2$	$n-1$
$mean(u)$	$2n^2 - n$	0%	$2u+1$	n
$is\_prime(n)^*$	1	0%	$(n-1)! + n + 3$	1
$fact(m)$	n	0%	m	n
$mult(u)$	$(n-1)! + 2$	0%	$u+1$	$(n-1)! + 2$
$app1(n_1, n_2, n_3)^*$	n_1	0%	$\mathcal{O}(n_1n_2n_3)^\dagger$	1
$app2(m_1, m_2)$	n_1n_2	0%	m_1m_2	n_1
$app3(u)$	$2n_1n_2n_3$	0%	u	$n_1n_2 + n_1$
$dyade(n_1, n_2)^*$	n_1	0%	$n_1(n_2+1)$	1
$mult(m)$	n_1n_2	0%	m	n_1
$minsort(n)^*$	$n+1$	0%	$\frac{(n+1)^2}{2} + \frac{n+1}{2}$	1
$findmin(m)$	$\frac{(n+1)^2}{2} + \frac{n-1}{2}$	7%	m	$n+1$
$hanoi(n)^*$	$2^n - 1$	0%	$2^{n+1} - 2$	1
$move(m)$	$2^n - 1$	0%	1	$2^n - 1$
$coupled(n)^*$	1	0%	$n+2$	1
$p(m)$	$\frac{n}{2} + \frac{(-1)^n}{4} + \frac{3}{4}$	1.2%	$m+1$	$\frac{n}{2} - \frac{(-1)^n}{4} + \frac{1}{4}$
$q(u)$	$\frac{n}{2} - \frac{(-1)^n}{4} + \frac{1}{4}$	0%	$u+1$	$\frac{n}{2} + \frac{(-1)^n}{4} - \frac{1}{4}$
$search(n)^*$	1	0%	$2n+2$	1
$member(m)$	$2n+1$	0%	$2m+1$	$2n+1$
$sublist(n_1, n_2)^*$	$n_2 + 3$	5%	$n_1n_2 + 3n_2 + 2$	2
$append(m)$	$n_1n_2 + 2n_2 - 1$	40%	$2m-1$	$n_1n_2 + 2n_2 - 1$

Experimental Results: Times (milliseconds)

Cost-Center	**Accumulated Cost UB** Cost Relations	Transformation (FLOPS'16)	**Standard Cost UB**	**Acc / Std**
*variance** *sq_diff* *mean*	3283 (-45%)	6038	3066	1.07
*isprime** *fact* *mult*	1245 (-42%)	2172	1231	1.01
*app1** *app2* *app3*	4150 (-34%)	6328	3757	1.11
*minsort** *findmin*	3400 (-29%)	4845	3300	1.03
*dyade** *mult*	3097 (-24%)	4117	2853	1.08
*hanoi** *move*	1605 (-19%)	1996	1376	1.16
*coupled** *f* *g*	2407 (-14%)	3112	1877	1.28
*search** *member*	1079	N/A	1071	1.00
*sublist** *append*	3674	N/A	3610	1.01
Average	**2652 (-33%)**	**4125**	**2542**	**1.05**

Tools / timeline

'83	Parallel abstract machines → motivation: auto-parallelization.
'88	**MA3 analyzer**: memo tables (cf. OLDT resolution), practicality established.
'89	**PLAI analyzer**: accelerated fixpoint, abstract domains as plugins. Sharing analysis, side-effect analysis.
90's	Incremental analysis, concurrency (dynamic scheduling), automatic domain combinations, scalability, auto-parallelization, extension to constraints.
'90	**GraCos analyzer**: fully automatic cost analysis (upper bounds).
early 90's	Automatic parallelization with task granularity control.
mid 90's	*CiaoPP model: Integrated verification/debugging/optimization w/assertions.*
'97-present	**CiaoPP tool**:
'91-'06	Combined **abstract interpretation and partial evaluation**.
late 90's	**Lower bounds** cost analysis. Non-failure (no exceptions), determinacy.
'01	**Verification** of cost, additional resources, ...
'01-05	Modularity/scalability. Diagnosis (locating origin of assrt. violations). New shape/type domains, widenings. Polyhedra, convex hulls.
'03	Abstraction carrying code, reduced certificates.
'04	Verification/debugging/optimization of **user-defined** resources.
'05	**Multi-language support** using CLP as IR: Java, C# (shapes, resources, ...).
'08	Verification of exec. **time**. First results in **energy** (Java), heap models, ...
'12	(X)C program energy analysis/verification, ISA-level energy models.
'13	**Cost anal. as Abs. Int.** Sized shapes. LLVM. **Accumulated Cost**

Thank you!

Experimental Results

Prog.	Resource An. (LB)			Resource An. (UB)					An. Time (s)	
	New	*Prev.*		*New*	*Prev.*		*RAML*		*New*	*Prev.*
`append`	α	α	$=$	β	β	$=$	β	$=$	1.00	0.53
`appAll`	$a_1 a_2 a_3$	a_1	$+$	$b_1 b_2 b_3$	∞	$+$	$b_1 b_2 b_3$	$=$	2.41	0.67
`coupled`	μ	0	$+$	ν	∞	$+$	ν	$=$	1.37	0.64
`dyade`	$\alpha_1\alpha_2$	$\alpha_1\alpha_2$	$=$	$\beta_1\beta_2$	$\beta_1\beta_2$	$=$	$\beta_1\beta_2$	$=$	1.66	0.62
`erathos`	α	α	$=$	β^2	β^2	$=$	β^2	$=$	2.25	0.77
`fib`	ϕ^μ	ϕ^μ	$=$	ϕ^ν	ϕ^ν	$=$	infeas.	$+$	1.06	0.67
`hanoi`	1	0	$+$	2^ν	∞	$+$	infeas.	$+$	0.82	0.60
`isort`	α^2	α^2	$=$	β^2	β^2	$=$	β^2	$=$	1.68	0.62
`isortl`	a_1^2	a_1^2	$=$	$b_1^2 b_2$	∞	$+$	$b_1^2 b_2$	$=$	2.55	0.67
`lisfact`	$\alpha\gamma$	α	$+$	$\beta\delta$	∞	$+$	unkn.	?	1.39	0.64
`listnum`	μ	μ	$=$	ν	ν	$=$	unkn.	?	1.19	0.58
`minsort`	α^2	α	$+$	β^2	β^2	$=$	β^2	$=$	1.94	0.67
`nub`	a_1	a_1	$=$	$b_1^2 b_2$	∞	$+$	$b_1^2 b_2$	$=$	3.61	0.91
`part`	α	α	$=$	β	β	$=$	β	$=$	1.70	0.65
`zip3`	$\min(\alpha_i)$	0	$+$	$\min(\beta_i)$	∞	$+$	β_3	$+$	2.48	0.57

IR Issues: Approaches to Performing the Transformation

- The transformation (akin to *Abstract Compilation*):
  - Source: Program P in L_P + (possibly abstract) Semantics of L_P
  - Target: A (C) Horn Clause program capturing $[\![P]\!]$ (or, possibly, $[\![P]\!]^\alpha$)

- Some approaches to performing the transformation:
  - Partial evaluation of instrumented interpreters + slicing.
    - Systematic construction from small- and big-step semantics.
    - Correctness proof more direct.
    - Not always fully automatic?
  - Direct transformation into block-based intermediate representation.
    - More control but correctness proof more indirect.
    - Used in the following (translation to a Ciao program).
    - Can add assertions to help analysis (sizes, metrics, resource models, ..).

The two approaches can produce similar results.

CFG traversal

- Blocks are nodes; edges are invocations.
- Top-down traversal of this CFG, starting from entry point.
- Within each block: sequence of builtins, handled in the domain.
- Inter-block calls/edges: *project*, *extend*, etc. (next slide).
- As graph is traversed, triples $(block, \lambda_{in}, \lambda_{out})$ are stored for each block in a *memo table*.
- Memo table entries have status $\in \{fixpoint, approx., complete\}$.
- Iterate until all *complete*.

Interprocedural analysis / recursion support

- **Project** the caller state over the actual parameters,
- find all the **compatible implementations** (blocks),
- **rename** to their formal parameters,

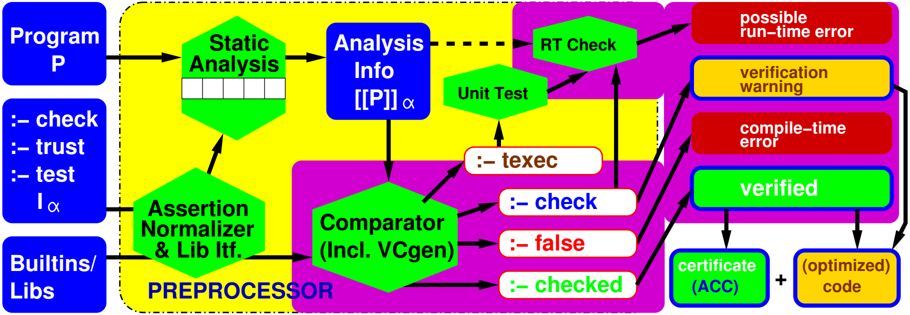
... abstractly execute each compatible block, ...

- calculate the **least upper bound** of the partial results of each block (if "monovariant on success" flag),
- **rename back** to the actual parameters and, finally
- **extend** (reconcile) return state into calling state.

Speeding up convergence

- Analyze non-recursive blocks first,
  use as starting λ_{out} in recursions.
- Blocks derived from conditionals treated specially
  (no *project* or *extend* operations required).
- The $(block, \lambda_{in}, \lambda_{out})$ tuples
  act as a cache that avoids recomputation.
- Use strongly-connected components (on the fly).

Integrated Static/Dynamic Debugging and Verification



Program P

:– check
:– trust
:– test
I_α

Builtins/ Libs

Static Analysis

Assertion Normalizer & Lib Itf.

PREPROCESSOR

Analysis Info $[[P]]_\alpha$

Comparator (Incl. VCgen)

RT Check

Unit Test

:– texec

:– check

:– false

:– checked

possible run–time error

verification warning

compile–time error

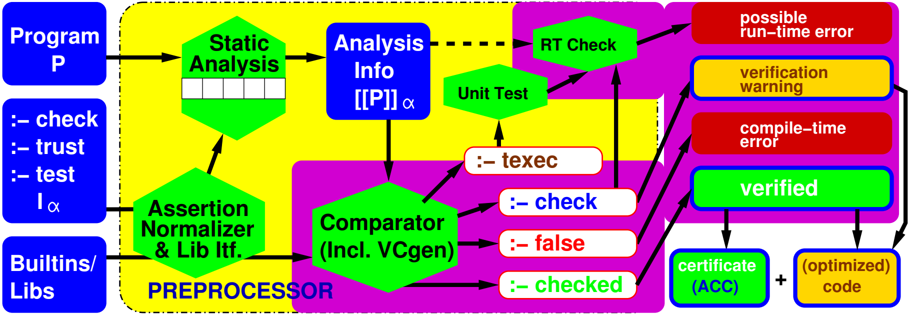
verified

certificate (ACC) + **(optimized) code**

	Definition	Sufficient condition
P is prt. correct w.r.t. $\mathcal{I}_\alpha$ if	$\alpha(\llbracket P \rrbracket) \leq \mathcal{I}_\alpha$	$\llbracket P \rrbracket_{\alpha^+} \leq \mathcal{I}_\alpha$
P is complete w.r.t. $\mathcal{I}_\alpha$ if	$\mathcal{I}_\alpha \leq \alpha(\llbracket P \rrbracket)$	$\mathcal{I}_\alpha \leq \llbracket P \rrbracket_{\alpha^=}$
P is incorrect w.r.t. $\mathcal{I}_\alpha$ if	$\alpha(\llbracket P \rrbracket) \not\leq \mathcal{I}_\alpha$	$\llbracket P \rrbracket_{\alpha^=} \not\leq \mathcal{I}_\alpha$, or $\llbracket P \rrbracket_{\alpha^+} \cap \mathcal{I}_\alpha = \emptyset \wedge \llbracket P \rrbracket_\alpha \neq \emptyset$
P is incomplete w.r.t. $\mathcal{I}_\alpha$ if	$\mathcal{I}_\alpha \not\leq \alpha(\llbracket P \rrbracket)$	$\mathcal{I}_\alpha \not\leq \llbracket P \rrbracket_{\alpha^+}$

[BDD$^+$97, HPB99, PBH00c, PBH00a, HPBLG03, HALGP05, PCPH06, PCPH08, MLGH09, SMH14, SMH15, SMH16]

Integrated Static/Dynamic Debugging and Verification



- Based throughout on the notion of *safe approximation* (abstraction).
- Run-time checks generated for *parts* of asserts. not verified statically.
- Diagnosis (for both static and dynamic errors).
- Comparison not always trivial: e.g., resource debugging/certification
  - Need to compare functions.
  - “Segmented” answers.

[BDD+97, HPB99, PBH00c, PBH00a, HPBLG03, HALGP05, PCPH06, PCPH08, MLGH09, SMH14, SMH15, SMH16]

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