Program Verification using Constraint Handling Rules and Array Constraint Generalizations

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Proof of Partial Correctness: An Example

Consider a **program** and a **partial correctness triple** for the program:

```
prog: while(x < n) {
        x = x+1 ;
        y = y+2 ;
      }
```

$$\{\, x=0 \wedge y=0 \wedge n \geq 1 \,\}\ \ prog\ \ \{y > x\}$$

(A) Generate the **Verification Conditions** (VC's)

1.	$x=0 \wedge y=0 \wedge n \geq 1 \rightarrow P(x,y,n)$	Initialization
2.	$P(x,y,n) \wedge x < n \rightarrow P(x+1,y+2,n)$	Loop
3.	$P(x,y,n) \wedge x \geq n \rightarrow y > x$	Exit

(B) Then, prove **satisfiability** of the VC's.

If the VC's are **satisfiable**, then the **partial correctness triple** holds.

Proof of Satisfiability of VC's

VC's are **satisfiable** if there is an interpretation that makes them true.

In our case,

$$P(x,y,n) \equiv (x=0 \wedge y=0 \wedge n \geq 1) \vee y > x$$

makes the VC's true. Indeed,

1'. $x=0 \wedge y=0 \wedge n \geq 1 \rightarrow (x=0 \wedge y=0 \wedge n \geq 1) \vee y > x$

2'. $((x=0 \wedge y=0 \wedge n \geq 1) \vee y > x) \wedge x < n$
$\rightarrow (x+1=0 \wedge y+2=0 \wedge n \geq 1) \vee y+2 > x+1$

3'. $((x=0 \wedge y=0 \wedge n \geq 1) \vee y > x) \wedge x \geq n \rightarrow y > x$

Thus, $\{x=0 \wedge y=0 \wedge n \geq 1\}$ *prog* $\{y > x\}$ holds.

- How to ***automatically*** prove **satisfiability** of the VC's?

Automatic Proofs of Satisfiability of VC's

Various methods:

- CounterExample Guided Abstraction Refinement (CEGAR), Interpolation, Satisfiability Modulo Theories [Rybalchenko et al., McMillan, Alberti et al.]
- Symbolic execution of Constraint Logic Programs [Jaffar et at.]
- Static Analysis and Transformation of Constraint Logic Programs [Gallagher et al., Albert et al.]

Our CLP Transformation Method

(A) Generate the VC's as a **constraint logic program (a CLP program)**:

V: 1*. p(X,Y,N) :- X=0, Y=0, N ≥ 1. (a constrained fact)

2*. p(X1,Y1,N) :- X<N, X1=X+1, Y1=Y+2, p(X, Y, N).

3*. incorrect :- X≥N, Y≤X, p(X, Y, N).

THM: The VC's are **satisfiable** iff incorrect ∉ **the least model** $M(V)$.

(B) Apply transformation rules that **preserve the least model** $M(V)$.

V': 4. q(X1, Y1, N) :- X<N, X>Y, Y≥0, X1=X+1, Y1=Y+2, q(X, Y, N).

5. incorrect :- X≥N, Y≤X, Y≥0 N≥1, q(X, Y, N).

least model preserved: incorrect ∉ $M(V)$ iff incorrect ∉ $M(V')$

no constrained facts for q: incorrect ∉ $M(V')$

Thus, $\{x=0 \wedge y=0 \wedge n\geq 1\}$ *prog* $\{y>x\}$ holds.

Outline

(A) How to generate the VC's, i.e., V?
(B) How to prove satisfiability of the VC's, i.e., transform V into V'?

Basic ideas from [PEPM-13].

Rules and strategies for programs on integers and integer arrays.

- CLP program transformation:
  - Unfold/fold rules: preserving the least model
  - (A) Strategies for VC's generation:
    specialization of the interpreter (getting V)
  - (B) Strategies for VC's satisfiability proof:
    propagation of constraints (getting V')
- Running example: Ascending Array Initialization, e.g., [3, 4, 5, 6]
- Experimental evaluation.

Rules for Transforming CLP Programs

R1. **Definition**. Introducing a new predicate (e.g., a loop invariant)

```
newp(X) :- c,A.
```

R2. **Unfolding**. A symbolic evaluation step (e.g., a resolution step)

given `H :- c, A, G.`
`A :- d1, G1. , ..., A :- dm,Gm.`

derive `H :- c,d1,G1,G. , ..., H :- c,dm,Gm,G.`

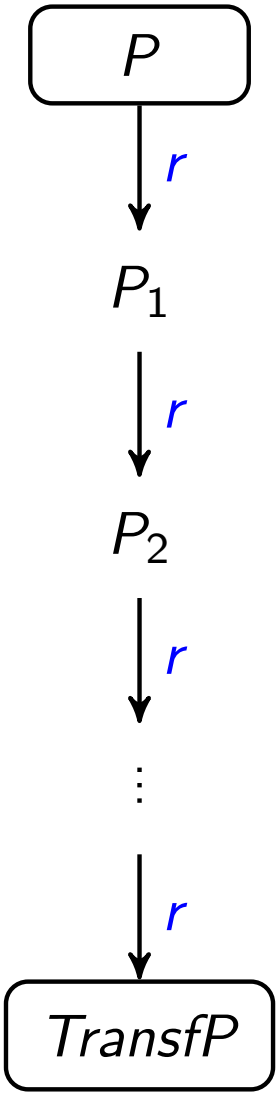
R3. **Folding**. Using a predicate definition

given `H :- d, A, G.`
`newp(X) :- c, A.` and $d \rightarrow c$

derive `H :- d, newp(X), G.`

R4. **Clause Removal**. Delete clauses with
(i) unsatisfiable constraint or (ii) subsumed by other clauses

'Rule + Strategies' Program Transformation



- The transformation **rules**:

  $r \in \{$ Definition, Unfolding, Folding, Clause Removal $\}$

- The rules **preserve** the least model:

Theorem (Least model preservation)

$\texttt{incorrect} \in M(P)$ iff $\texttt{incorrect} \in M(\mathit{TransfP})$

- The rules must be guided by **strategies**.

[Burstall-Darlington 77, Tamaki-Sato 84, Etalle-Gabbrielli 96]

Encoding Partial Correctness into CLP

Consider the triple $\{\varphi_{init}\}\ prog\ \{\neg\varphi_{error}\}$.

A program *prog* is incorrect w.r.t. φ_{init} and φ_{error}
if a final configuration satisfying φ_{error}
is reachable from an initial configuration satisfying φ_{init}.

Definition (the interpreter *Int* with the transition predicate `tr(X,Y)`)

```
reach(X) :- initConf(X).
reach(Y) :- tr(X,Y), reach(X).
incorrect :- errorConf(X), reach(X).
```

+ clauses for `tr` (i.e., the operat. semantics of the programming language)

Theorem

prog is incorrect iff `incorrect` $\in M(Int)$

A program *prog* is correct iff it is not incorrect.

tr(X,Y): the operational semantics

`L: Id = Expr`	`tr( cf(cmd(L,asgn(Id,Expr)),S), cf(cmd(L1,C1),S1)) :-` `  aeval(Expr,S,V),` *evaluate expression* `  update(Id,V,S,S1),` *update store* `  nextlabel(L,L1),` *next label* `  at(L1,C1).` *next command*

`L: if(Expr) {`
`  L1: ...`
`}`
`else`
`  L2: ...`
`}`
	`tr( cf(cmd(L,ite(Expr,L1,L2)),S), cf(C,S)) :-`
	`  beval(Expr,S),` *expression is true*
	`  at(L1,C).` *next command*
	`tr( cf(cmd(L,ite(Expr,L1,L2)),S), cf(C,S)) :-`
	`  beval(not(Expr),S),` *expression is false*
	`  at(L2,C).` *next command*

`L: goto L1`	`tr( cf(cmd(L,goto(L1)),S), cf(C,S)) :-` `  at(L1,C).` *next command*

tr(X,Y): the operational semantics for array assignment

array assignment: L : a[ie] = e

```
tr( cf(cmd(L,asgn(elem(A,IE),E)),S),   source configuration cf
    cf(cmd(L1,C),S1) ) :-              target configuration cf
      eval(IE,S,I),                    evaluate index expr IE
      eval(E,S,V),                     evaluate expression E
      lookup(S,array(A),FA),           get array FA from store
      write(FA,I,V,FA1),               update array FA, getting FA1
      update(S,array(A),FA1,S1),       update store S, getting S1
      nextlab(L,L1),                   next label L1
      at(L1,C).                        command C at next label
```

Running Example: Ascending Array Initialization

Given the **program** *SeqInit* and the **partial correctness triple**

```
i=1;
while(i<n) {
 a[i]=a[i-1]+1;
 i=i+1;
}
```

$$\{i \geq 0 \wedge n \geq 1 \wedge n = dim(\mathtt{a})\}$$
$$SeqInit$$
$$\{\forall j\ (0 \leq j \wedge j+1 < n \ \rightarrow\ \mathtt{a}[j] < \mathtt{a}[j+1])\}$$

CLP encoding of program *SeqInit*

- A set of $\mathtt{at}(\mathtt{label}, \mathtt{command})$ facts.
- $\mathtt{while} = \mathtt{ite} + \mathtt{goto}$.
- $\mathtt{elem}(\mathtt{a},\mathtt{i})$ stands for $\mathtt{a}[\mathtt{i}]$.

$\mathtt{at}(\ell_0, \mathtt{asgn}(\mathtt{i}, 1)))$.
$\mathtt{at}(\ell_1, \mathtt{ite}(\mathtt{less}(\mathtt{i},\mathtt{n}), \ell_2, \ell_h))$.
$\mathtt{at}(\ell_2, \mathtt{asgn}(\mathtt{elem}(\mathtt{a},\mathtt{i}),$
$\quad \mathtt{plus}(\mathtt{elem}(\mathtt{a}, \mathtt{minus}(\mathtt{i}, 1)), 1)))$.
$\mathtt{at}(\ell_3, \mathtt{asgn}(\mathtt{i}, \mathtt{plus}(\mathtt{i}, 1)))$.
$\mathtt{at}(\ell_4, \mathtt{goto}(\ell_1))$.
$\mathtt{at}(\ell_h, \mathtt{halt})$.

CLP encoding of φ_{init} and φ_{error}

$\mathtt{initConf}(\ell_0, \mathtt{I}, \mathtt{N}, \mathtt{A})$:-
$\mathtt{I} \geq 0,\ \mathtt{N} \geq 1$.

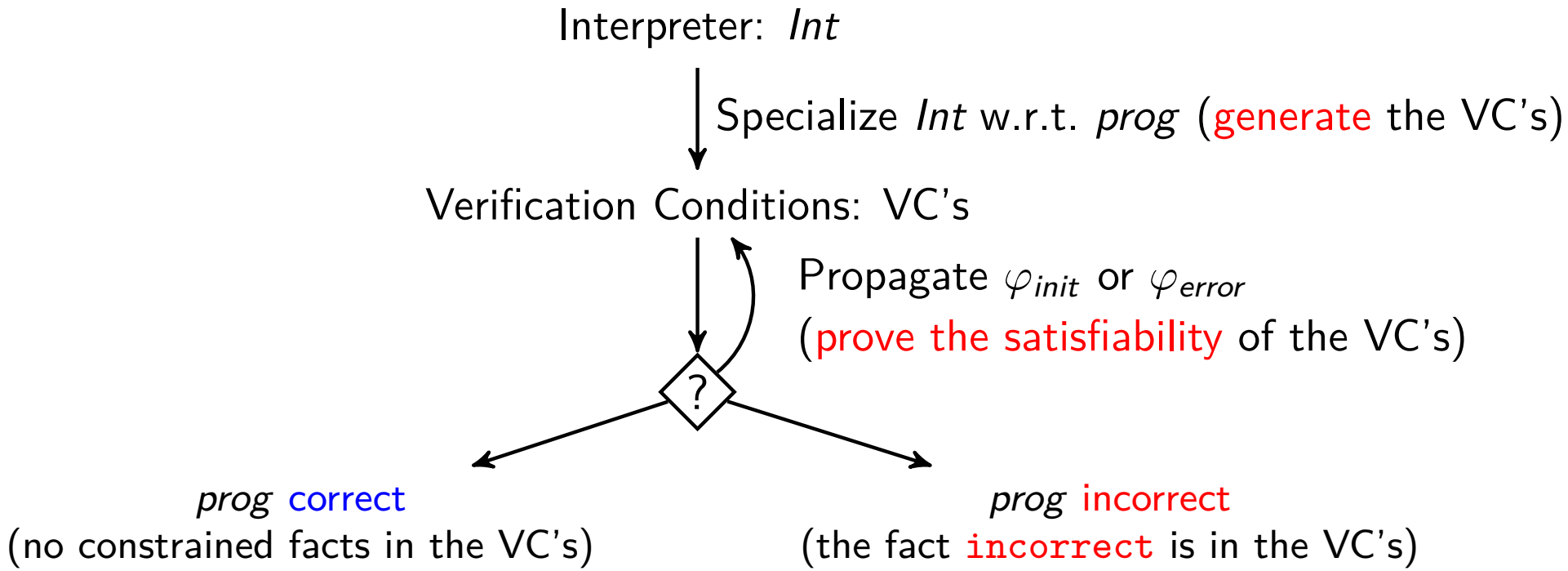
$\mathtt{errorConf}(\ell_h, \mathtt{N}, \mathtt{A})$:-
$\mathtt{W} \geq 0,\ \mathtt{W}+1 < \mathtt{N},\ \boxed{\mathtt{Z} = \mathtt{W}+1,\ \mathtt{U} \geq \mathtt{V},}$
$\mathtt{read}(\mathtt{A}, \boxed{\mathtt{W}, \mathtt{U}}), \mathtt{read}(\mathtt{A}, \boxed{\mathtt{Z}, \mathtt{V}})$.

The Transformation-based Verification Method

Program Transformation of CLP is used to

(A) generate the VC's

(B) prove the satisfiability of the VC's



The Strategy for Generation

Transform(P)

$TransfP = \emptyset$;

Defs $= \{$`incorrect :- errorConf(X), reach(X)`$\}$;

while $\exists q \in$ Defs **do**

```
% execute a symbolic evaluation step (i.e., resolution)
```

Cls = Unfolding(q);

```
% remove unsatisfiable and subsumed clauses
```

Cls = ClauseRemoval(Cls);

```
% introduce new predicates (i.e., a loop invariant)
```

Defs = (Defs $- \{q\}$) $\cup$ Definition(Cls);

```
% match a predicate definition
```

$TransfP = TransfP \cup$ Folding(Cls, Defs);

od

Verification Conditions Generation

The specialization of *Int* w.r.t. *prog* removes all references to:

- `tr` and
- `at`

VC's: the Specialized Interpreter for *SeqInit*

```
incorrect :- Z=W+1, W≥0, W+1<N, U≥V, N≤I,
             read(A,W,U), read(A,Z,V), new1(I,N,A).
new1(I1,N,B) :- 1≤I, I<N, D=I-1, I1=I+1, V=U+1,
             read(A,D,U), write(A,I,V,B), new1(I,N,A).
new1(I,N,A) :- I=1, N≥1.
```

- A constrained fact is present:
  we cannot conclude that the program is correct.
- The fact `incorrect` is not present:
  we cannot conclude that the program is incorrect either.

The Strategy for Satisfiability

Transform(P)

$TransfP = \emptyset$;

Defs $= \{$`incorrect :- errorConf(X), reach(X)`$\}$;

while $\exists q \in$ Defs **do**

  Cls $=$ Unfolding(q);

  Cls $=$ ConstraintReplacement(Cls);

  Cls $=$ ClauseRemoval(Cls);

  Defs $=$ (Defs $- \{q\}) \cup$ $\text{Definition}_{\text{array}}$(Cls);

  $TransfP = TransfP \cup$ Folding(Cls, Defs);

od

Constraint Replacement Rule

If $\mathcal{A} \models \forall (c_0 \leftrightarrow (c_1 \vee \ldots \vee c_n))$, where $\mathcal{A}$ is the Theory of Arrays

Then replace `H :- c0, d, G`
by `H :- c1, d, G, ..., H :- cn, d, G`

Constraint Handling Rules for Constraint Replacement:

AC1. Array-Congruence-1: if i=j then a[i]=a[j]

$$\texttt{read(A,I,X)} \setminus \texttt{read(A1,J,Y)} \Leftrightarrow \texttt{A} == \texttt{A1}, \texttt{I} = \texttt{J} \mid \texttt{X} = \texttt{Y}.$$

AC2. Array-Congruence-2: if a[i]≠a[j] then i≠j

$$\texttt{read(A,I,X)}, \texttt{read(A1,J,Y)} \Rightarrow \texttt{A} == \texttt{A1}, \texttt{X} <> \texttt{Y} \mid \texttt{I} <> \texttt{J}.$$

ROW. Read-Over-Write: {a[i]=x; y=a[j]} if i=j then x=y

$$\texttt{write(A,I,X,A1)} \setminus \texttt{read(A2,J,Y)} \Leftrightarrow \texttt{A1} == \texttt{A2} \mid (\texttt{I} = \texttt{J}, \texttt{X} = \texttt{Y}) \; ; \; (\texttt{I} <> \texttt{J}, \texttt{read(A,J,Y)}).$$

Ascending Array Initialization

```
new3(A,B,C) :- A=2+H, B-H≤3, E-H≤1, E≥1, B-H≥2,...,
  read(N,H,M), read(C,D,F), write(N,J,K,C), read(C,E,G),
  reach(J,B,N).
```

- by applying the ROW rule:

```
new3(A,B,C) :- J=1+D, A=2+D, K=1+I, I<F, ...,
  read(C,D,F), read(N,D,I), write(N,J,K,C), read(C,E,G),
  reach(J,B,N).
new3(A,B,C) :- J=1+D, A=2+D, K=1+I, I<F, ...,
  read(C,D,F), read(N,D,I), write(N,J,K,C), read(C,E,G),
  reach(J,B,N).
```

- by applying the ROW, AC1, and AC2 rules:

```
new3(A,B,C) :- A=1+H, E=1+D, J=-1+H, K=1+L, D-H≤-2, H<B,...
  read(N,E,G), read(N,D,F), read(N,J,L), write(N,H,K,C),
  reach(J,B,M).
```

Ascending Array Initialization

```
new3(A,B,C) :- A=2+H, B-H≤3, E-H≤1, E≥1, B-H≥2,...,
  read(N,H,M), read(C,D,F), write(N,J,K,C), read(C,E,G),
  reach(J,B,N).
```

- by applying the ROW rule:

```
new3(A,B,C) :- J=1+D, A=2+D, K=1+I, I<F, ..., J=E, K=G,
  read(C,D,F), read(N,D,I), write(N,J,K,C), ~~read(C,E,G)~~,
  reach(J,B,N).
new3(A,B,C) :- J=1+D, A=2+D, K=1+I, I<F, ...,
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- by applying the ROW, AC1, and AC2 rules:

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new3(A,B,C) :- A=1+H, E=1+D, J=-1+H, K=1+L, D-H≤-2, H<B,...
  read(N,E,G), read(N,D,F), read(N,J,L), write(N,H,K,C),
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Ascending Array Initialization

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  reach(J,B,N).
```

- by applying the ROW rule:

```
new3(A,B,C) :- J=1+D, A=2+D, K=1+I, I<F, ..., J=E, K=G,
  read(C,D,F), read(N,D,I), write(N,J,K,C), ~~read(C,E,G)~~,
  reach(J,B,N).
new3(A,B,C) :- J=1+D, A=2+D, K=1+I, I<F, ..., J<>E,
  read(C,D,F), read(N,D,I), write(N,J,K,C), read(C,E,G),
  reach(J,B,N).
```

- by applying the ROW, AC1, and AC2 rules:

```
new3(A,B,C) :- A=1+H, E=1+D, J=-1+H, K=1+L, D-H≤-2, H<B,...
  read(N,E,G), read(N,D,F), read(N,J,L), write(N,H,K,C),
  reach(J,B,M).
```

Ascending Array Initialization

```
new3(A,B,C) :- A=2+H, B-H≤3, E-H≤1, E≥1, B-H≥2,...,
  read(N,H,M), read(C,D,F), write(N,J,K,C), read(C,E,G),
  reach(J,B,N).
```

- by applying the ROW rule:

```
new3(A,B,C) :- J=1+D, A=2+D, K=1+I, I<F, ..., J=E, K=G,
  read(C,D,F), read(N,D,I), write(N,J,K,C), ~~read(C,E,G)~~,
  reach(J,B,N).
new3(A,B,C) :- J=1+D, A=2+D, K=1+I, I<F, ..., J<>E,
  read(C,D,F), read(N,D,I), write(N,J,K,C), read(C,E,G),
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- by applying the ROW, AC1, and AC2 rules:

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new3(A,B,C) :- A=1+H, E=1+D, J=-1+H, K=1+L, D-H≤-2, H<B,...
  read(N,E,G), read(N,D,F), read(N,J,L), write(N,H,K,C),
  reach(J,B,M).
```

Definition Introduction

Introduction of suitable new predicate **definitions** (they correspond to **program invariants**).

Difficulty: Introduction of an unbounded number of new predicate definitions.

Solution: Use of **generalization** operators:

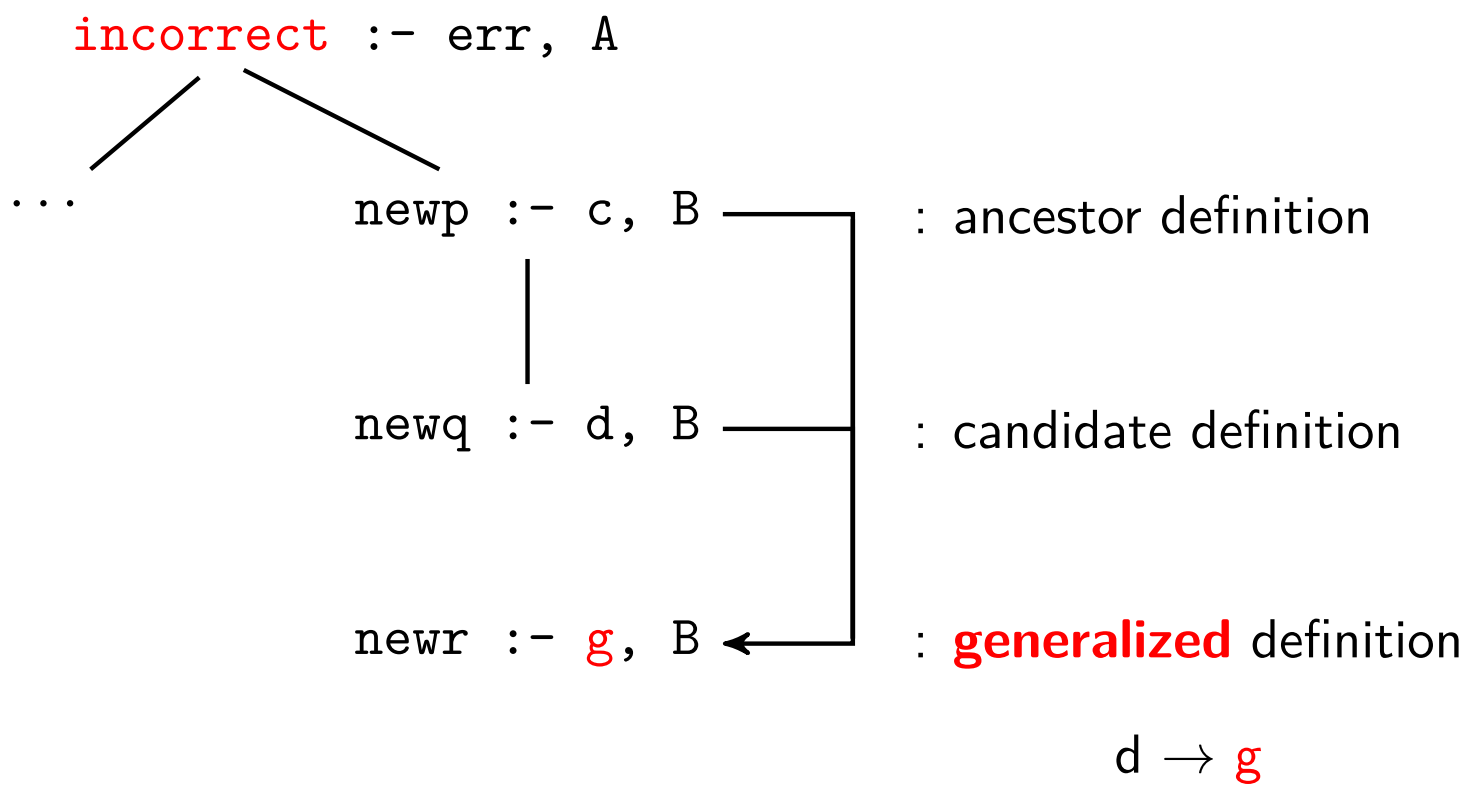
- to ensure the termination of the transformation,
- to generate program invariants.

Note. They are two somewhat conflicting requirements:

- (efficiency) introduction of as few definitions as possible, and
- (precision) proof of as many satisfiability properties as possible.

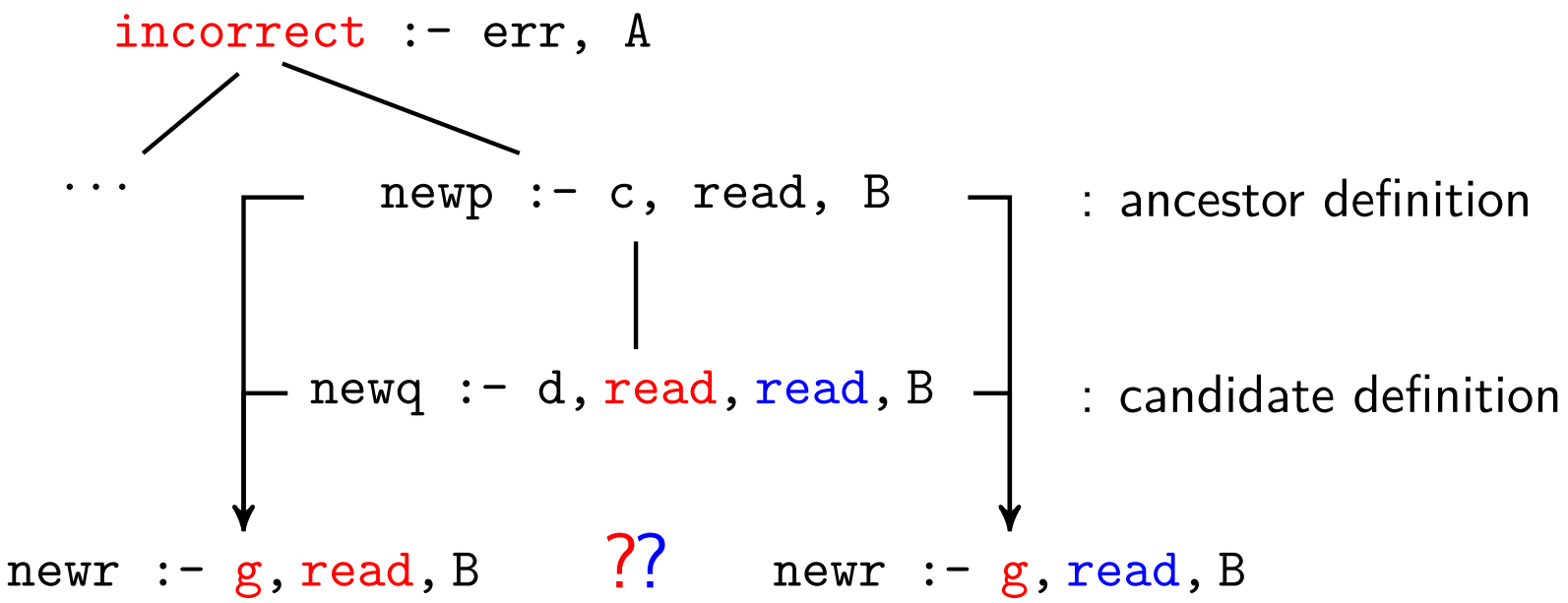
Constraint Generalizations

Definitions are arranged as a tree:



Generalization operators based on widening and convex-hull.

Array Constraint Generalizations



- We decorate CLP variables with the variable identifiers of the imperative program.

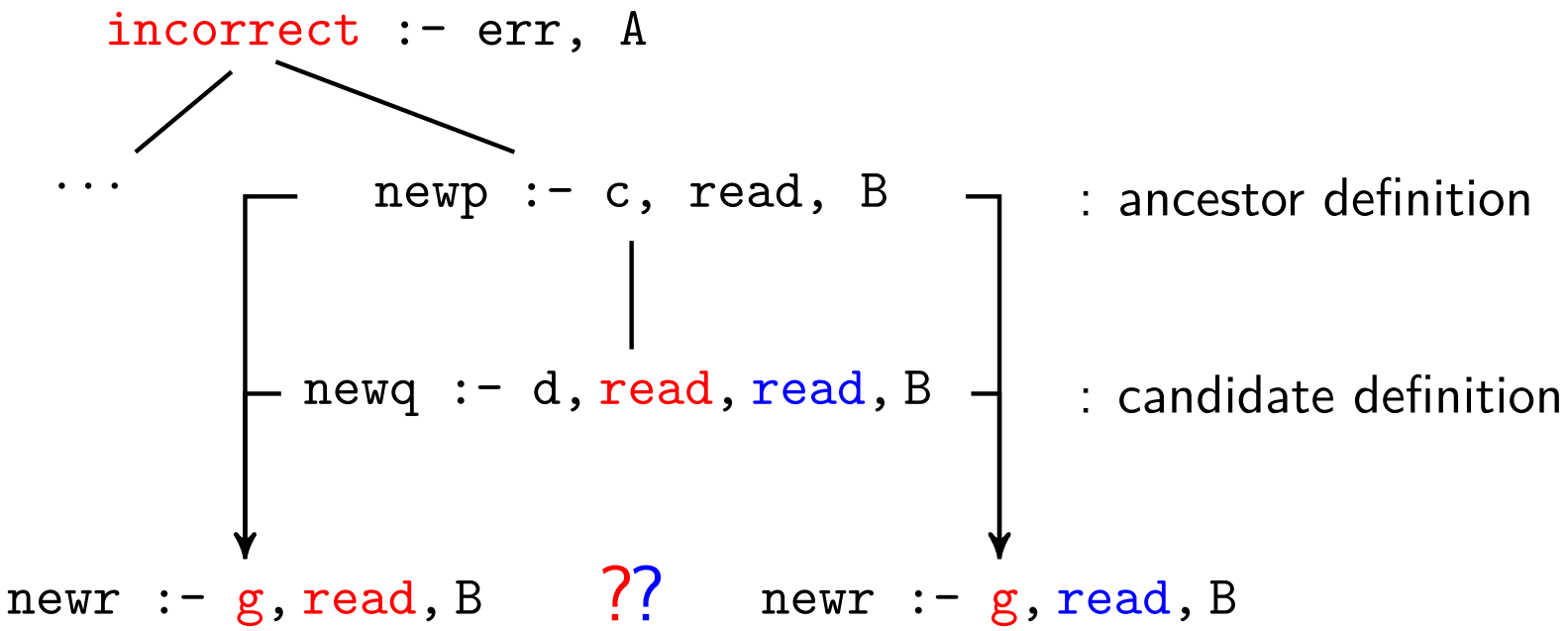
VC's: the Specialized Interpreter for *SeqInit*

$$\texttt{incorrect} \;\texttt{:-}\; \texttt{Z}=\texttt{W}+1,\ \texttt{W}\geq 0,\ \texttt{W}+1<\texttt{N},\ \texttt{U}\geq\texttt{V},\ \texttt{N}\leq\texttt{I},\ \texttt{read}(\texttt{A},\texttt{W}^{\texttt{j}},\texttt{U}^{\texttt{a[j]}}),\ \texttt{read}(\texttt{A},\texttt{Z}^{\texttt{j1}},\texttt{V}^{\texttt{a[j1]}}),\ \texttt{new1}(\texttt{I},\texttt{N},\texttt{A}).$$

$$\texttt{new1}(\texttt{I1},\texttt{N},\texttt{B}) \;\texttt{:-}\; 1\leq\texttt{I},\ \texttt{I}<\texttt{N},\ \texttt{D}=\texttt{I}-1,\ \texttt{I1}=\texttt{I}+1,\ \texttt{V}=\texttt{U}+1,\ \texttt{read}(\texttt{A},\texttt{D}^{\texttt{i}},\texttt{U}^{\texttt{a[i]}}),\ \texttt{write}(\texttt{A},\texttt{I},\texttt{V},\texttt{B}),\ \texttt{new1}(\texttt{I},\texttt{N},\texttt{A}).$$

$$\texttt{new1}(\texttt{I},\texttt{N},\texttt{A}) \;\texttt{:-}\; \texttt{I}=1,\ \texttt{N}\geq 1.$$

Array Constraint Generalizations



- We decorate CLP variables with the variable identifiers of the imperative program.

VC's: the Specialized Interpreter for *SeqInit*

```
incorrect :- Z=W+1, W≥0, W+1<N, U≥V, N≤I,
             read(A, W^j, U^a[j]), read(A, Z^j1, V^a[j1]), new1(I, N, A).
new1(I1, N, B) :- 1≤I, I<N, D=I−1, I1=I+1, V=U+1,
             read(A, D^i, U^a[i]), write(A, I, V, B), new1(I, N, A).
new1(I, N, A) :- I=1, N≥1.
```

Ascending Array Initialization

: ancestor definition

```
new3(I,N,A) :- E+1=F, E≥0, I>F, G≥H, N>F, N≤I+1,
read(A,E^j,G^a[j]), read(A,F^j1,H^a[j1]), reach(I,N,A).
```

: candidate definition

```
new4(I,N,A) :- E+1=F, E≥0, I>F, G≥H, I=1+I1, I1+2≤C, N≤I1+3,
read(A,E^j,G^a[j]), read(A,F^j1,H^a[j1]), read(A,P^i,Q^a[i]),
reach(I,N,A).
```

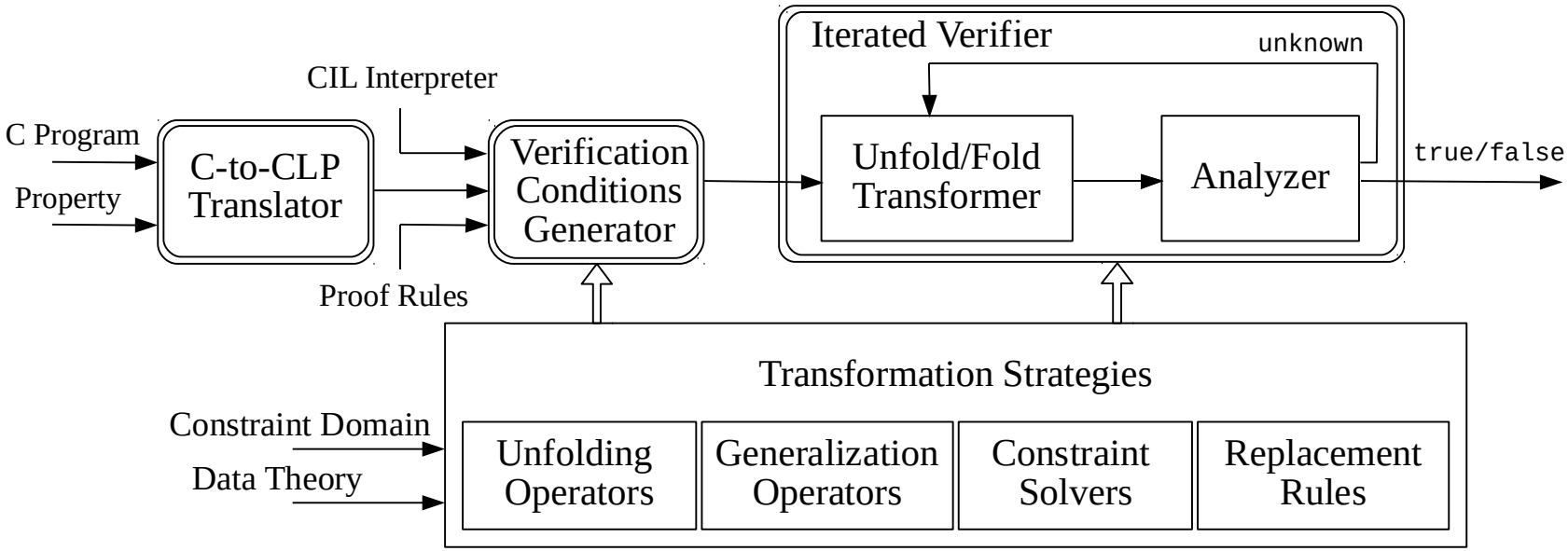
: **generalized** definition

```
new5(I,N,A) :- E+1=F, E≥0, I>F, G≥H, N>F,
read(A,E^j,G^a[j]), read(A,F^j1,H^a[j1]), reach(I,N,A).
```

In the paper: a variable of the form G^v is encoded by `val(v,G)`.

VeriMAP

- The VeriMAP tool `http://map.uniroma2.it/VeriMAP`



Experimental evaluation

Program	$Gen_{W,\mathcal{I},\Cap}$	$Gen_{H,\mathcal{V},\subseteq}$	$Gen_{H,\mathcal{V},\Cap}$	$Gen_{H,\mathcal{I},\subseteq}$	$Gen_{H,\mathcal{I},\Cap}$
bubblesort-inner	0.9	*unknown*	*unknown*	*unknown*	1.52
copy-partial	*unknown*	*unknown*	3.52	3.51	3.54
copy-reverse	*unknown*	*unknown*	5.25	*unknown*	5.23
copy	*unknown*	*unknown*	5.00	4.88	4.90
find-first-non-null	0.14	0.66	0.64	0.28	0.27
find	1.04	6.53	2.35	2.33	2.29
first-not-null	0.11	0.22	0.22	0.22	0.22
init-backward	*unknown*	1.04	1.04	1.03	1.04
init-non-constant	*unknown*	2.51	2.51	2.47	2.47
init-partial	*unknown*	0.9	0.89	0.9	0.89
init-sequence	*unknown*	4.38	4.33	4.41	4.29
init	*unknown*	1.00	0.97	0.98	0.98
insertionsort-inner	0.58	2.41	2.4	2.38	2.37
max	*unknown*	*unknown*	0.8	0.81	0.82
partition	0.84	1.77	1.78	1.76	1.76
rearrange-in-situ	*unknown*	*unknown*	3.06	3.01	3.03
selectionsort-inner	*unknown*	*time-out*	*unknown*	2.84	2.83
precision	6	10	15	15	17
total time	3.61	21.42	34.76	31.81	38.45
average time	0.60	2.14	2.31	2.12	2.26

Conclusions and Future Work

- Parametric verification framework (semantics, logics, constraint domains)
  - CLP as a metalanguage
  - agile way of synthesizing software verifiers [Rybalchenko et al.]
- Semantics preserving transformations
  - iterative verification
  - use Horn clauses for passing information between verifiers [McMillan]
- **Future work**
  - more experiments (including programs with nested loops)
  - more theories (lists, heaps, etc.)
  - Other programming languages and properties.